# RITS USULI YORDAMIDA IKKINCHI TARTIBLI CHIZIQLI CHEGARAVIY MASALANI TAQRIBIY YECHISH

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*Annotatsiya: Mazkur maqolada ikkinchi tartibli chiziqli differensial tenglama uchun qo'yilgan chegaraviy masala Rits variatsion usuli yordamida taqribiy yechildi. Tenglama o'z-o'ziga qo'shma ko'rinishga keltirilib, integrallovchi ko'paytuvchi aniqlandi. Chegaraviy shartlarni qanoatlantiruvchi boshlang'ich va bazis funksiyalar tanlandi hamda Rits usuliga mos algebraik tenglamalar sistemasi tuzildi. Hisoblashlar natijasida taqribiy yechimning analitik ifodasi olindi.*

*Kalit so'zlar: Rits usuli, variatsion usullar, chegaraviy masala, differensial tenglama, sonli usullar.*

Kirish. Amaliy matematika va muhandislik masalalarida uchraydigan ko'plab jarayonlar differensial tenglamalar yordamida modellashtiriladi. Biroq ko'plab hollarda bunday tenglamalarni analitik usullar bilan aniq yechish imkoni mavjud emas. Shu sababli taqribiy va sonli usullardan foydalanish zarur bo'ladi.

Rits (Ritz) usuli variatsion prinsipga asoslangan bo'lib, asosan chegaraviy masalalarni yechishda qo'llaniladi. Ushbu usul yechimni oldindan tanlangan bazis funksiyalar chiziqli kombinatsiyasi ko'rinishida izlashga asoslanadi. Mazkur maqolada Rits usuli yordamida aniq bir differensial tenglama uchun chegaraviy masalani yechish bosqichma-bosqich ko'rib chiqiladi.

Masalaning matematik qo'yilishi. Misol: Rits metodi bilan yeching.

$$y'' + 2xy' + 3y = 1.5$$

chegaraviy shartlar bilan:

$$y(0.6) = 1.1$$
$$y(0) = 2$$

bu yerda:

$$p(x) = 2x, q(x) = 3, f(x) = 1.5$$

tenglamani o'z-o'ziga qo'shma ko'rinishga keltirish Rits usulini qo'llash uchun tenglama o'z-o'ziga qo'shma ko'rinishda yoziladi. Integrallovchi ko'paytuvchi quyidagicha aniqlanadi:

$$\rho(x) = e^{\int_a^x p(t)dt}$$
$$Q(x) = \rho(x)q(x)$$
$$F(x) = \rho(x)p(x)$$

$$\frac{\partial}{\partial x}\left(\rho(x)y'\right) + Q(x)y = F(x)$$

$$\rho(x) = e^{\int_{0.6}^x 2tdt} = e^{x^2-0.36}$$

misolimiz quyidagi ko‘rinishga keladi:

$$\frac{\partial}{\partial x}\left(e^{x^2-0.36}y'\right) + 3e^{x^2-0.36}y = 1.5e^{x^2-0.36}$$

$$y(0.6) = 1.1$$
$$y(0) = 2$$

boshlang‘ich funksiya Chegaraviy shartlarni qanoatlantiruvchi chiziqli funksiya tanlanadi:

$$\varphi_0(x) = 2.5x - 0.5$$
$$\varphi_i(x) = (x-a)(x-b)^i$$
$$\varphi_1(x) = (x-0.6)(x-1)^1$$
$$\varphi_2(x) = (x-0.6)(x-1)^2$$

Rits tenglamalar sistemasini tuzish. Rits usuliga ko‘ra quyidagi sistema hosil qilinadi:

$$y = \varphi_0(x) + \sum_{i=1}^{n} a_i\varphi_i(x)$$

$$\sum_{i=1}^{n} a_i A_{ij} = b_j$$

bu yerda:

$$A_{ij} = \int_a^b \left[\rho(x)\varphi_i'(x)\varphi_j'(x) + Q(x)\varphi_i(x)\varphi_j(x)\right]dx$$

$$b_j = \int_a^b \left[\rho(x)\varphi_0'(x)\varphi_j'(x) + Q(x)\varphi_0(x)\varphi_j(x) + \varphi_j(x)F(x)\right]dx$$

$$A_{11} = \int_{0.6}^{1} \left[\rho(x)\varphi_1'(x)\varphi_1'(x) + Q(x)\varphi_1(x)\varphi_1(x)\right]dx =$$

$$\int_{0.6}^{1}\left[e^{x^2-0.36}(2x-1.6)^2+3e^{x^2-0.36}(x^2-1.6x+0.6)^2\right]dx=0.031$$

$$A_{21}=\int_a^b\left[\rho(x)\varphi_2'(x)\varphi_1'(x)+Q(x)\varphi_2(x)\varphi_1(x)\right]dx=$$

$$=\int_{0.6}^{1}\left[e^{x^2-0.36}(3x^2-5.2x+2.2)(2x-1.6)+3e^{x^2-0.36}(x^3-2.6x^2+2.2x-0.6)(x^2-1.6x+0.6)\right]dx$$

$$=-0.005$$

$$A_{12}=\int_a^b\left[\rho(x)\varphi_1'(x)\varphi_2'(x)+Q(x)\varphi_1(x)\varphi_2(x)\right]dx=$$

$$=\int_{0.6}^{1}\left[e^{x^2-0.36}(2x-1.6)(3x^2-5.2x+2.2)+3e^{x^2-0.36}(x^3-2.6x^2+2.2x-0.6)(x^2-1.6x+0.6)\right]dx$$

$$=-0.005$$

$$A_{22}=\int_a^b\left[\rho(x)\varphi_2'(x)\varphi_2'(x)+Q(x)\varphi_2(x)\varphi_2(x)\right]dx=$$

$$=\int_{0.6}^{1}\left[e^{x^2-0.36}(3x^2-5.2x+2.2)^2+3e^{x^2-0.36}(x^3-2.6x^2+2.2x-0.6)^2\right]dx=0.00016$$

$$b_1=\int_a^b\left[\rho(x)\varphi_0'(x)\varphi_1'(x)+Q(x)\varphi_0(x)\varphi_1(x)+\varphi_1(x)F(x)\right]dx=$$

$$=\int_{0.6}^{1}\left[e^{x^2-0.36}2.5(2x-1.6)+3e^{x^2-0.36}(2.5x-0.5)(x^2-1.6x+0.6)+(x^2-1.6x+0.6)1.5e^{x^2-0.36}\right]dx$$

$$=-0.029$$

$$b_2=\int_a^b\left[\rho(x)\varphi_0'(x)\varphi_2'(x)+Q(x)\varphi_0(x)\varphi_2(x)+\varphi_2(x)F(x)\right]dx=$$

$$=\int_{0.6}^{1}\left[e^{x^2-0.36}2.5(3x^2-5.2x+2.2)+3e^{x^2-0.36}(2.5x-0.5)(x^3-2.6x^2+2.2x-0.6)+(x^3-2.6x^2+2.2x-0.6)1.5e^{x^2-0.36}\right]dx$$

$$=0.0051$$

$$A_{11}=0.031,\ A_{21}=-0.005,\ A_{12}=-0.005,\ A_{22}=0.00016$$

$$b_1=-0.029,\ b_2=0.0051$$

quyidagi sistemani yechish orqali $a_1$ va $a_2$ larni topamiz.

$$\begin{aligned}A_{11}a_1+A_{21}a_2&=b_1\\A_{12}a_1+A_{22}a_2&=b_2\end{aligned}$$

$$a_1 = -0.85, a_2 = 0.53$$

topilgan $a_1$ va $a_2$ larni yechimga olib borib qo‘ysak quyidahi ko‘rinishga keladi

Taqribiy yechim

Topilgan koeffitsiyentlarni yechimga qo‘ysak, quyidagi taqribiy funksiya hosil bo‘ladi:

$$y = 2.5x - 0.5 - 0.85(x-0.6)(x-1) + 0.53(x-0.6)(x-1)^2$$

ILOVALAR

```
import numpy as np
# Simpson integration
def simpson(f, a, b, n=1000):
    if n % 2 == 1:
        n += 1
    h = (b - a) / n
    s = f(a) + f(b)
    for i in range(1, n):
        x = a + i * h
        s += (4 if i % 2 == 1 else 2) * f(x)
    return s * h / 3
# Functions
def rho(x):
    return np.exp(x**2 - 0.36)
def Q(x):
    return 3 * rho(x)


def F(x):
    return 1.5 * rho(x)
# Basis functions
def phi0(x):
    return 2.5 * x - 0.5
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```
def phi1(x):
    return (x - 0.6) * (x - 1)
def phi2(x):
    return (x - 0.6) * (x - 1)**2
# Derivatives of basis functions
def phi0p(x):
    return 2.5
def phi1p(x):
    return 2 * x - 1.6
def phi2p(x):
    return 3 * x**2 - 5.2 * x + 2.2
# Integration intervals
a, b = 0.6, 1.0
# A_ij matrix elements
A11 = simpson(lambda x: rho(x) * phi1p(x)**2 + Q(x) * phi1(x)**2, a, b)
A12 = simpson(lambda x: rho(x) * phi1p(x) * phi2p(x) + Q(x) * phi1(x) * phi2(x),
a, b)
A22 = simpson(lambda x: rho(x) * phi2p(x)**2 + Q(x) * phi2(x)**2, a, b)

# b_i vector elements
b1 = simpson(lambda x: rho(x) * phi0p(x) * phi1p(x) + Q(x) * phi0(x) * phi1(x) +
phi1(x) * F(x), a, b)
b2 = simpson(lambda x: rho(x) * phi0p(x) * phi2p(x) + Q(x) * phi0(x) * phi2(x) +
phi2(x) * F(x), a, b)
# Matrix and vector
A = np.array([[A11, A12],
[A12, A22]])
B = np.array([b1, b2])
# Solving the linear system
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```
a1, a2 = np.linalg.solve(A, B)
# Final solution
def y(x):
    return 2.5 * x - 0.5 + a1 * (x - 0.6) * (x - 1) + a2 * (x - 0.6) * (x - 1)**2
# Output results
print("a1 =", a1)
print("a2 =", a2)
print("y(0.6) =", y(0.6))
print("y(1.0) =", y(1.0))
```

## XULOSA

Ushbu maqolada Rits variatsion usuli yordamida ikkinchi tartibli chiziqli differensial tenglama uchun chegaraviy masala to‘liq yechildi. Differensial tenglama o‘z-o‘ziga qo‘shma ko‘rinishga keltirildi va Rits usuli asosida algebraik tenglamalar sistemasi tuzildi. Olingan taqribiy yechim chegaraviy shartlarni aniq qanoatlantiradi va usulning amaliy samaradorligini ko‘rsatadi.

## FOYDALANILGAN ADABIYOTLAR

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