CHIZIQSIZ DIFERENSIAL TENGLAMALAR UCHUN RUNGE-KUTTA USULLARINING MODIFIKATSIYALARI.

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Annotatsiya: Ushbu maqolada chiziqsiz differensial tenglamalarni sonli yechishda keng qo'llaniladigan Runge–Kutta usullarining turli modifikatsiyalari o'rganiladi. Klassik Runge–Kutta usullarining chiziqsiz masalalarda yuzaga keladigan aniqlik, barqarorlik va hisoblash samaradorligi bilan bog'liq muammolari tahlil qilinadi. Shu asosda adaptiv qadamli Runge–Kutta usullari, implicit (yashirin) Runge–Kutta sxemalari, shuningdek yuqori tartibli va o'zgartirilgan koeffitsiyentli modifikatsiyalar ko'rib chiqiladi. Har bir modifikatsiyaning afzalliklari va kamchiliklari chiziqsiz differensial tenglamalar misolida taqqoslab baholanadi. Tadqiqot natijalari murakkab chiziqsiz tizimlarni modellashtirishda Runge–Kutta usullarining mos modifikatsiyasini tanlash imkonini beradi hamda amaliy masalalarni yechishda hisoblash aniqligi va barqarorligini oshirishga xizmat qiladi.

Kalit so'zlar: Chiziqsiz differensial tenglamalar, Runge–Kutta usuli, Runge–Kutta usullarining modifikatsiyalari, sonli usullar, adaptiv qadam usuli, implicit Runge–Kutta sxemalari, barqarorlik, aniqlik, matematik modellashtirish, hisoblash algoritmlari.

KIRISH. Zamonaviy hisoblash texnikasi va yig'ilgan hisoblash tajribalari differensial tenglamalarning katta va murakkab masalalarini taqribiy yechish imkonini bermoqda. Sonli hisoblashlarda eng muhim jihat bu yetarlicha aniqlikda izlanayotgan taqribiy yechimga erishishdir. Buning uchun esa oddiy differensial tenglamalarni taqribiy yechishning hisoblash usullari va ularning xususiyatlari bilan yaqindan tanishishni talab qiladi. Bu bilan birga shunday masalalar ham uchraydiki, ularni mavjud usullar bilan emas, balki ularning modifikatsiyasi, yangi uslubi va algoritmi bilan yechish lozim bo'ladi. Umuman olganda, oddiy differensial tenglama bilan berilgan chegaraviy masala: yagona yechimga ega; yechimga ega emas; bir nechta yoki cheksiz ko'p yechimga ega bo'lisi mumkin. Koshi masalasini yechish usullari: Teylor qatori yordamida approksimatsiyalash, Runge-Kutta usullari, tahlil va korreksiya usuli va boshqalar.

Agar tenglamada noma'lum funksiya hosila yoki differensial ostida qatnashsa, bunday tenglama differensial tenglama deyiladi. Agar differensial tenglamada noma'lum funksiya faqat bir o'zgaruvchiga bog'liq bo'lsa, bunday tenglama oddiy differensial tenglama deyiladi. Masalan:

$$\frac{dy}{dx}=\sqrt{3}(1-2y);\ \ y'=\frac{x^4}{2};\ \ \frac{dy}{dt}=t^2-1;\ \ \sqrt{2}\frac{d^2y}{dx^2}=x^2+1;\ \ xdy=3dx;$$

Agar differensial tenglamadagi noma`lum funksiya ikki yoki undan ortiq o'zgaruvchilarga bog'liq bo'lsa, bunday tenglama xususiy hosilali differensial tenglama deyiladi. Masalan:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f(x,y,z)$$

differensial tenglamaning tartibi deb, shu tenglamada qatnashuvchi hosilaning (differensialning) eng yuqori tartibiga aytiladi. Masalan:

$$(u')^3=x^2+2$$

birinchi tartibli tenglamalar,

$$\frac{\partial^4 u}{\partial x^4}=5(\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 u}{\partial z^3}), \qquad \frac{\partial^4 T}{\partial t^4}=1-(l^t+2)$$

esa 4-tartibli differensial tenglamalardir.Mavzularda faqat oddiy differensial tenglamalarni ko'rib chiqamiz. n –tartibli oddiy differensial tenglamaning umumiy ko'rinishi quyidagicha:

$$F(x,y,y',y'',.......,y^{(n)})=0 \tag{1.1.1}$$

bu yerda x – erkli o'zgaruvchi; y – noma`lum funksiya, $y,y',y'',.......,y^{(n)}$ - noma`lum funksiyaning hosilalari.

(1.2.1) ni ko'p hollarda quyidagi ko'rinishda yozish mumkin:

$$y^{(n)}=f(x,y,y',y'',.......,y^{(n-1)})=0 \tag{1.1.2}$$

(1.1.2) ning yechimi (yoki integrali) deb uni qanoatlantiruvchi shunday $y=\varphi(x)$ funksiyaga aytiladiki, $\varphi(x)$ ni (1.1.2) ga qo'yganda u ayniyatga aylanadi. Oddiy differensial tenglama yechimining grafigi uning integral egri chizig'i deyiladi.

Quyidagi chegaraviy masalani qaraymiz:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(p(U)\frac{\partial U}{\partial x}\right)+f(U),0<x<l,\ 0<t<T$$

$$U(x,0)=U_0(x),\ U(0,\mathrm{t})=\mu_1(t),U(1,\mathrm{t})=\mu_2(t) \qquad (1.2.1)$$

Odatda, chiziqli bo‘lmagan tenglamalarda $\rho(u)$ funksiyaning o‘zgarish sohasi oldindan ma’lum bo‘lmasa, oshkor sxemalar ishlatilmaydi.

Sof oshkormas sxema $y_i^{k+1}\left(i=\overline{1,M-1}\right)$ noma’lumlarga nisbatan chiziqli sistemani ham, chiziqli bo‘lmagan sistemani ham tashkil etishi mumkin. Ushbu sxema

$$\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{1}{h}\left[a_{i+1}\frac{y_{i+1}^{k+1}-y_i^{k+1}}{h}-a_i\frac{y_i^{k+1}-y_{i-1}^{k+1}}{h}\right]+f\left(y_i^k\right) \qquad (1.2.2)$$

da $a_i=\frac{1}{2}\left[\rho\left(y_i^k\right)+\rho\left(y_{i-1}^k\right)\right]$ deb olsak, u holda $y_i^{k+1}\left(i=\overline{1,M-1}\right)$ noma’lumlarga nisbatan chiziqli, absolyut turg‘un bo‘lib, approksimasiya xatoligi $r=0\left(\tau+h^2\right)$ bo‘ladi. Bu sistemaning yechimi haydash metodi bilan topiladi.

Ko‘pincha (1.2.1) tenglama uchun ushbu

$$\frac{y_i^{k+1}-y_i^k}{\tau}=\frac{1}{h}\left[a\left(y_{i+1}^{k+1}\right)\frac{y_{i+1}^{k+1}-y_i^{k+1}}{h}-a\left(y_i^{k+1}\right)\frac{y_i^{k+1}-y_{i-1}^{k+1}}{h}\right]+f\left(y_i^{k+1}\right)$$

$$a\left(y_i^{k+1}\right)=\frac{\rho\left(y_i^{k+1}\right)+\rho\left(y_{i-1}^{k+1}\right)}{2} \qquad (1.2.3)$$

sof oshkormas sxema ishlatiladi. Bu sxemani qo‘llash uchun u yoki bu iterasion metod qo‘llaniladi. Masalan, iteratsion jarayonni quyidagicha olib borishimiz mumkin:

$$\frac{y_i^{(S+1)}-y_i^k}{\tau}=\frac{1}{h}\left[a\left(y_{i+1}^{(S)}\right)\frac{y_{i+1}^{(S+1)}-y_i^{(S+1)}}{h}-a\left(y_i^{(S)}\right)\frac{y_i^{(S+1)}-y_{i-1}^{(S+1)}}{h}\right]+f\left(y_i^{(S)}\right)$$

$$S=0,1,\ldots,L-1,\ y_i^{(0)}=y_i^k,\ y_i^{(L)}=y_i^{k+1} \qquad (1.2.4)$$

bu erda S — iteratsiya nomeri. Bu iteratsion jarayondan ko‘ramizki, chiziqli bo‘lmagan koeffisiyentlar oldingi iteratsiyada, ya’ni y_i^k da hisoblanadi, y_i^{k+1} ning dastlabki yaqinlashishi sifatida y_i^k olinadi. Agar τ qadam qancha kichik bo‘lsa, bu dastlabki yaqinlashish shuncha yaxshi bo‘ladi. Agar koeffisiyentlar silliq bo‘lib,

$\rho(u) \ge C_2 > 0$ shart bajarilsa, odatda, ikki-uchta iteratsiya qoniqarli natijaga olib keladi. Har bir yangi iteratsiyada $y_i^{(S+1)}$ ning qiymatlari (1.2.4) sistemadan haydash metodi bilan aniqlanadi. Shuningdek, (1.2.4) sistemani yechish uchun ikkinchi tartibli aniqlikka ega bO'lgan *prediktor-korrektor* sxemasi ham ishlatiladi. Bunda k-qatlamdan $(k+1)$ qatlamga o'tish ikki bosqichda bajariladi. Birinchi bosqichda haydash metodi bilan oshkormas chiziqli sistema

$$\frac{y_i^{k+\frac{1}{2}} - y_i^k}{0,5\tau} = \frac{1}{h}\left[a\left(y_{i+1}^k\right)\frac{y_{i+1}^{k+\frac{1}{2}} - y_i^{k+\frac{1}{2}}}{h} - a\left(y_i^k\right)\frac{y_i^{k+\frac{1}{2}} - y_{i-1}^{k+\frac{1}{2}}}{h}\right] + f\left(y_i^k\right),\ i = 1,2,\dots,M-1,$$

$$y_0^{k+\frac{1}{2}} = \mu_1\left(t_k + 0,5\tau\right),\ y_M^{k+\frac{1}{2}} = y_2\left(t_k + 0,5\tau\right)$$

yechilib, oradagi $y_i^{k+\frac{1}{2}}(i = 0,1,\dots,M)$ qiymatlar topiladi. Ikkinchi bosqichda esa $a(y)$, $f(y)$ chiziqli bo'lmagan koeffisiyentlar $y = y_i^{k+\frac{1}{2}}$ da hisoblanib, y_i^{k+1} larni topish quyidagi olti nuqtali simmetrik sxema

$$\frac{y_i^{k+1} - y_i^k}{2} = \frac{1}{2h}\left[a\left(y_{i+1}^{k+\frac{1}{2}}\right)\frac{y_{i+1}^{k+1} - y_i^{k+1}}{h} - a\left(y_i^{k+\frac{1}{2}}\right)\frac{y_{i+1}^k - y_i^k}{h}\right] + f\left(y_i^{k+\frac{1}{2}}\right),$$

$$i = 1,2,\dots,M-1,\ y_0^{k+1} = \mu_1\left(t_{k+1}\right),\ y_M^{k+1} = \mu_2\left(t_{k+1}\right)$$

asosida olib boriladi.

Umumiy tushunchalar. Quyidagi

$$u' = f(x,u), u(x_0) = u_0 \qquad (1.3.1)$$

Koshi masalasining aniq yechimini $u(x)$ orqali belgilaymiz.Qaralayotgan sohada $f(x,u)$ yetarlicha silliq funksiya bo'lsin, u holda

$$u(x_1) - u(x_0) = \sum_{k=1}^{s} \frac{h^k}{k!} u^{(k)}(x_0) + 0(h^{s+1}) \qquad (1.3.2)$$

$$(x_1 = x_0 + h, h > 0)$$

Endi $u(x_1)$ ning taqribiy qiymatini u_1 orqali belgilab (2) tenglikda qoldiq hadni tashlasak,

$$\nabla u_0 = u_1 - u_0 = \sum_{k=1}^{s} \frac{h^k}{k!} u^{(k)}(x_0) \tag{1.3.3}$$

Yoyilma xosil bo‘ladi.Bu yoyilmadagi $u'(x_0), u''(x_0), \ldots$ xosilalar (1.3.1) tenglikdan aniqlanadi.Keyingi xisoblashlarga qulaylik tug‘dirish uchun ushbu operatorlarni kiritamiz:

$$\begin{aligned} D &= \frac{\partial}{\partial x} + f\frac{\partial}{\partial u} \\ D^2 &= \frac{\partial^2}{\partial x^2} + 2f\frac{\partial^2 u}{\partial x \partial u} + f^2\frac{\partial^2}{\partial u^2} \\ D^3 &= \frac{\partial^3}{\partial x^3} + 3f\frac{\partial^3}{\partial x^2 \partial u} + 3f\frac{\partial^3}{\partial x \partial^3 u} + f^3\frac{\partial^3}{\partial u^3} \end{aligned} \tag{1.3.4}$$

Bu yerda $f = f(x,u)$ (1.3.1) tenglamaning o‘ng tomoni. Bu operatorlar uchun quyidagi tengliklar orinlidir:

$$\begin{aligned} &D(y+z) = Dy + Dz \\ &D(yz) = zDy + yDz \\ &D(Dz) = D^2 z + Dz\frac{\partial z}{\partial u} \\ &D(D^2 z) = D^3 z + 2DfD\left(\frac{\partial z}{\partial u}\right) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &D(D^n(z)) = D^{m+1}(z) + mD(f)D^{m+1}\left(\frac{\partial z}{\partial u}\right) \end{aligned} \tag{1.3.5}$$

Misol. Barcha natural son $m \geq 2$ sonlar uchun (1.3.5) tenglik isbot qilinsin.

Murakkab funksiyani diferensialllash qoidasini qo‘llab, (1.3.1) tenglamadan va (1.3.4) tengliklardan ketma – ket quyidagilarni topamiz.

$$\begin{aligned} &u' = f, \\ &u'' = \frac{\partial f}{\partial x} + f\frac{\partial f}{\partial u} = Df \end{aligned} \tag{1.3.6}$$

$$u''' = D(Df) = D^2 f + \frac{\partial f}{\partial u} Df \tag{1.3.7}$$

$$u'' = D(D^2 f + \frac{\partial f}{\partial u} Df) = D(D^2 f) + D(\frac{\partial f}{\partial x})Df + \frac{\partial f}{\partial x} D(Df) =$$

$$= D^3 f + 2DfD(\frac{\partial f}{\partial x}) + DfD(\frac{\partial f}{\partial x}) + \frac{\partial f}{\partial x}(D^2 f + \frac{\partial f}{\partial u} Df) =$$

$$= D^3 f + \frac{\partial f}{\partial u} D^2 f + (\frac{\partial f}{\partial u})^2 Df + 3Df * D(\frac{\partial f}{\partial u})$$

Bu tengliklarning o‘ng tomoni (x,u) nuqtada xisoblangan deb qaraymiz. Shunday qilib (1.3.3) yoyilmadagi barcha u(x) xosilalarni nazariy jihatdan xisoblash mumkin. Ammo (1.3.6) formulalar noqulay va katta bo‘lganligi sababli ularni ∇u_0 ni toppish uchun amalyotda bevosita qo‘llash mushkuldir.

Runge ∇u_0 ni hisoblash uchun (1.3.3) ning o‘rnida p_n o‘zgarmas koifsentlar bilan olingan

$$k_j(h) = hf(\xi_j, n_i)(i = 1, 2, ..., r) \qquad (1.3.8)$$

funksiyalarning

$$\nabla u_0 = p_r, k_1(h) + p_{r2} k_2(h) + ... + p_{rr} k_r(h) \qquad (1.3.9)$$

chiziqli konbinasiyasini olishni taklif etdi, bu yerda

$$\xi_i = x_0 + a_i h, a_1 = 0,$$

$$\eta_i = u_0 + \beta_{i1} k_2(h) + ... + \beta_{i,r-1} k_{r-1}(h)$$

va a_i, β_{ij}-o‘zgarmas sonlardir. Shunday qilib

$$\left.\begin{aligned} &k_1(h) = hf(x_0, u_0), \\ &k_2(h) = hf(x_0 + a_2 h, u_0 + \beta_{21} k_1), \\ &k_3(h) = hf(x_0 + a_3 h, u_0 + \beta_{31} k_1 + \beta_{32} k_2), \\ &.. \\ &k_r(h) = hf(x_0 + a_r h, u_0 + \beta_{r1} k_1 + ... + \beta_{r,r-1} k_{r-1}(h)), \end{aligned}\right\} \qquad (1.3.10)$$

Bu yerda a_i, β_{ij} lar ma'lum bo'lsa, h ni tanlab, ketma-ket k(h0 larni hisoblash mumkin. p_{ri}, x_i, β_{ij} parametrlarni shunday tanlanganki, ixtiyoriy f(x,u) funksiya va ixtiyoriy h qadam uchun (1.3.3) va (1.3.7) yoyilmalarda h ning imkoni boricha yuqori darajasigacha bo'lgan xadlar ustma-ust tushsin. Boshqacha aytganda,

$$\varphi_r(h) = u(x_1) - u_0 - \sum_{j=1}^{r} p_{ri} k_i(h)$$

funksiya

$$\varphi_0(0) = \varphi_r'(0) = ... = \varphi^s{}_0(0) = 0, \varphi_r^{(s+1)}(0) \neq 0$$

xossalarga ega bo'lib p_{ri}, x_i, β_{ij} lar shunday tanlanishi kerakki, ixtiyoriy h va f(x,u) uchun s mumkin qadar katta bo'lsin. Runge-Kutta metodining xatoligi, ya'ni $u(x_1) - u_0$ bilan (1.3.9) formula yordamida xisoblangan uning taqribiy qiymati orasidagi farq har bir qadamda

$$R_r(h) = \frac{h^{(s+1)} \varphi_r^{(s+1)(0)}(\xi)}{(s+1)!}, 0 \leq \xi \leq h \tag{1.3.11}$$

ga tengdir. Bu yerda s- Runge-Kutta metodining aniqlik tartibi.(1.3.9) ko'rinishdagi formulalar Runge-Kutta formulalari deyiladi.Metodning asosiy g'oyasi Runge (1895) tomonidan taklif etilgan bo'lib, keyinchalik birinchi tartibli tenglama uchun Xeyn(1900) va Kutta (1901) yanada takomillashtirdilar, Nistrem, Sirmol quyida bu metodning ayrim xususiy xollarini ko'rib chiqamiz. Bu metodning umumiy xollarini [7,13] dan qarash mumkin.

Runge–Kuttaning oshkor usullari sinfi, Eylerning oshkor usullari hamda to'rtinchi tartibli Runge–Kutta usullarining umumlashgan ko'rinishi hisoblanadi. Ushbu metod quyidagi formula orqali beriladi:

$$y_{n+1} = y_n + h\sum_{i=1}^{s} b_i k_i,$$

bu yerda h — to'r qadamining x bo'yicha kattaligi. Yangi qiymatni hisoblash esa quyidagi s bosqichlarda amalga oshiriladi:

$$
\begin{aligned}
&k_1 = f(x_n, y_n),\\
&k_2 = f(x_n + c_2 h, y_n + a_{21} h k_1),\\
&\ldots\\
&k_s = f(x_n + c_s h, y_n + a_{s1} h k_1 + a_{s2} h k_2 + \ldots + a_{s,s-1} h k_{s-1})
\end{aligned}
$$

Aniq metod, s soni va b_i, a_{ij} hamda c_i koeffitsiyentlar orqali aniqlanadi. Ushbu koeffitsiyentlar Butcher jadvali deb ataluvchi jadvalni hosil qiladi:

$$
\begin{array}{cccccc}
0 & & & & & \\
c_2 & a_{21} & & & & \\
c_3 & a_{31} & a_{32} & & & \\
\mathrm{M} & \mathrm{M} & \mathrm{M} & \mathrm{O} & & \\
c_s & a_{s1} & a_{s2} & \cdots & a_{ss-1} & \\
 & b_1 & b_2 & \cdots & b_{s-1} & b_s
\end{array}
$$

Runge–Kutta usuli koeffitsiyentlari uchun $\sum_{j=1}^{i-1} a_{ij} = c_i$ va $i = 2,\ldots,s$ shart bajarilishi kerak. Agar metod aniqligi p -tartibli bo‘lishi kerak bo‘lsa, qo‘shimcha tarzda quyidagi shart ham bajarilishi lozim:

$$\overline{\mathbf{y}}(h + x_0) - \mathbf{y}(h + x_0) = O(h^{p+1}),$$

bu yerda $\overline{\mathbf{y}}(h + x_0)$ — Runge–Kutta usuli orqali olingan yaqinlashish.

Ko‘p martalab differensiallashdan so‘ng ushbu shart, metod koeffitsiyentlariga nisbatan polinomial tenglamalar sistemasiga aylanadi. Sistemani quyidagi ko‘rinishda ifodalash mumkin:

$$
\begin{aligned}
&\mathrm{P}_1(\mathrm{x}, \mathrm{y}) = 0\\
&\mathrm{P}_2(\mathrm{x}, \mathrm{y}) = 0\\
&\ldots\\
&\mathrm{P}_n(\mathrm{x}, \mathrm{y}) = 0
\end{aligned}
$$

bu yerda: P_1, $\mathrm{P}_2, \ldots,$ P_n-polinomial tenglamalar; x, y-o‘zgaruvchilar.

Polinomial tenglama quyidagi ko‘rinishga ega:

$$P(x) = a_n x^n + a_{(n-1)} x^{(n-1)} + \ldots + a_1 x + a_0 = 0$$

bu yerda - $\mathrm{P}(\mathrm{x})$ - polynomial funksiya: - $\mathrm{a}_n, \mathrm{a}_{n-1}, \ldots, \mathrm{a}_1, \mathrm{a}_0$ -polinom koeffitsiyentlari (bular haqiqiy yoki kompleks sonlar bo‘lishi mumkin); n-polinomning darajasi; x-o‘zgaruvchi. Masalan, uchinchi darajali polinomial tenglama ko‘rinishi quyidagi kabi bo‘lishi mumkin:

$$2x^3 - 4x^2 + 3x - 5 = 0$$

bu tenglamada: daraja n=3; koeffitsiyentlar: $a_3 = 2$, $a_2 = -4$, $a_1 = 3$, $a_0 = -5$. Polinomial tenglamalar har xil darajaga ega bo‘lishi mumkin va ularni yechish uchun turli usuldan foydalaniladi.

Ushbu

$$u^{(n)} = f(x, u, u', ..., u^{(n-1)}) \qquad (1.2.1)$$

n-tartibli oddiy differensial tenglamalar uchun ham Runge-Kutta metodlari ishlab chiqilgan. Ma’lumki, almashtirishlar bajarib, (1.2.1) tenglamasi differensial tenglamalar sistemasining normal shakliga keltirilishi mumkin. Biz yuqorida k (k = 1,2,3,4) tartibli Runge-Kutta metodining formulalarini chiqargan edik. Bu formulalarni bemalol tenglamalar sistemasi uchun ham qo‘llash mumkin.

Faraz qilaylik, ushbu

$$u' = f_1(x, u, z), \quad z' = f_2(x, u, z)$$

tenglamalar sistemasining

$$u(x_0) = u_0, \quad z(x_0) = z_0$$

dastlabki shartlarni qanoatlantiradigan yechimni topish talab qilinsin. Bitta tenglama bo‘lgan holga o‘xshab parallel ravishda $\Delta u_{0,} \Delta z_0$ sonlarini aniqlaymiz.

$$\left.\begin{aligned} \Delta u_0 &= \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4), \\ \Delta z_0 &= \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4) \end{aligned}\right\} \qquad (1.2.2)$$

bu yerda

$$\begin{aligned} k_1 &= hf_1(x_0, u_0, z_0), & l_1 &= hf_2(x_0, u_0, z_0), \\ k_2 &= hf_1(x_0 + \frac{h}{2}, u_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}), & l_2 &= hf_2(x_0 + \frac{h}{2}, u_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}), \\ k_3 &= hf_1(x_0 + \frac{h}{2}, u_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}), & l_3 &= hf_2(x_0 + \frac{h}{2}, u_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}), \\ k_4 &= hf_1(x_0 + h, u_0 + k_3, z_0 + l_3), & l_4 &= hf_2(x_0 + h, u_0 + k_3, z_0 + l_3). \end{aligned}$$

natijada

$$u_1 = u_0 + \Delta u_0, \qquad z_1 = z_0 + \Delta z_0$$

formulaga ega bo‘lamiz.

Misol: Quyidagi oddiy differensial tenglamani ko‘rib chiqamiz: $y' = y - x^2 + 1, \quad y(0) = 0.5$ Runge-Kutta metodi yoirdamida $h = 0.2$ qadam uzunligi bilan $x = 1$ gacha bo‘lgan yechimni topamiz. Har bir qadamdagi xatoliklarni hisoblash va tahlil qilamiz.

$$k_1 = f(x_n, y_n) = y_n - x_n^2 + 1 = 0.5 - 0^2 + 1 = 1.5$$

$$k_2 = f\left(0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} \cdot 1.5\right) = f(0.1, 0.65) = 0.65 - 0.1^2 + 1 = 1.65 \; k_3 = f\left(0.1, 0.65\right) = 1.65$$

$$k_4 = f(0.2, 0.83) = 0.83 - 0.2^2 + 1 = 1.79$$

$$y_1 = 0.5 + \frac{0.2}{6}(1.5 + 2 \cdot 1.65 + 2 \cdot 1.65 + 1.79) = 0.5 + \frac{0.2}{6} \cdot 9.24 = 0.875$$

Lokal xatolikni hisoblash uchun haqiqiy yechim $y(0.2)$ ni topamiz. Bu holda haqiqiy yechim $y(x) = e^x - x^2 - 1$ ga teng bo‘ladi:

$$y(0.2) = e^{0.2} - 0.2^2 - 1 \approx 1.2214 - 0.04 - 1 \approx 0.1814$$

Lokal xatolik: $y\left(0.2\right) - y_1 = | 0.1814 - 0.875 | = 0.6936$

$$k_1 = f(0.2, 0.875) = 1.795$$

$$k_2 = f(0.3, 1.0345) = 1.8145$$

$$k_3 = f(0.3, 1.0345) = 1.8145$$

$$k_4 = f(0.4, 1.2389) = 1.8789 \; y_2 = 0.875 + \frac{0.2}{6}(1.795 + 2 \cdot 1.8145 + 2 \cdot 1.8145 + 1.8789) = 1.1820$$

Haqiqiy yechim $y(0.4)$: $y(0.4) = e^{0.4} - 0.4^2 - 1 \approx 1.4918 - 0.16 - 1 \approx 0.3318$

Lokal xatolik: $y\left(0.4\right) - y_2 = | 0.3318 - 1.1820 | = 0.8502$

XULOSA. Mazkur maqolada chiziqsiz oddiy differensial tenglamalarni sonli yechish masalalari keng ko‘lamda o‘rganildi. Ishning asosiy maqsadi Runge–Kutta usullari va ularning modifikatsiyalarining nazariy asoslarini tahlil qilish hamda ushbu usullarni chiziqsiz differensial tenglamalarga qo‘llash orqali ularning samaradorligini amaliy misollar asosida baholashdan iborat bo‘ldi.

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