GIPERBOLIK TIPDAGI TENGLAMALAR SISTEMASI UCHUN GURSA TIPDAGI TO‘G‘RI MASALA

Xo‘jayev L.H., Saipnazarov J.M., Shoimova K.S.

QDTU dotsenti, f-m.f.f.d (PhD). lochin-x@mail.ru.

QDTU dotsenti, f-m.f.f.d (PhD). jonibeksaipnazarov@gmail.com.

Axborot texnologiyalari va menejment universiteti magistranti

Annotatsiya. Mazkur ishda giperbolik tipdagi tenglamalar sistemasi uchun Gursa tipidagi to‘g‘ri masala o‘rganiladi. Tadqiqotda masalaning matematik modeli tahlil qilinib, yechimning mavjudligi, yagonaligi va berilgan boshlang‘ich hamda chegaraviy shartlarga bog‘liqligi ko‘rib chiqiladi. Shuningdek, giperbolik tenglamalar sistemasining xossalari va ularning amaliy qo‘llanish imkoniyatlari tahlil qilinadi. Olingan natijalar matematik fizika, to‘lqin jarayonlari va elastik muhitdagi dinamik hodisalarni modellashtirishda muhim ahamiyatga ega.

Kalit so‘zlar. giperbolik tenglamalar sistemasi, Gursa masalasi, to‘g‘ri masala, boshlang‘ich shartlar, chegaraviy shartlar, yechimning mavjudligi, yagonalik, matematik fizika.

KIRISH. Giperbolik tipdagi tenglamalar matematik fizika va mexanikada to‘lqin uzatilishi, zarba tarqalishi va dinamik jarayonlarni tavsiflashda keng qo‘llaniladi. Ushbu tenglamalar asosida qo‘yilgan masalalardan biri — Gursa tipidagi to‘g‘ri masala, bunda tenglama sistemasi ma’lum boshlang‘ich va chegaraviy shartlar bilan beriladi. Gursa tipidagi masalalar giperbolik tizimlarning fizik jihatdan real va barqaror yechimlarini ta’minlash uchun muhimdir. Bunday masalalarda yechimning mavjudligi, yagonaligi va ma’lumotlarga uzluksiz bog‘liqligi matematik jihatdan tahlil qilinadi. Shu sababli, Gursa tipidagi masalalarni o‘rganish nafaqat nazariy matematikada, balki seysmologiya, elastik muhitdagi zarba tarqalishi, neft va gaz konlarini izlash kabi amaliy sohalarda ham ahamiyatga ega. Maqolada giperbolik tenglamalar sistemalari uchun Gursa tipidagi to‘g‘ri masalalar matematik modeli, yechim shartlari va barqarorlik xususiyatlari tahlil qilinadi, shuningdek, ularning amaliy qo‘llanilish imkoniyatlari ko‘rib chiqiladi.

MASALANI QO‘YILISHI VA YECHISH METODIKASI. $\tilde{D}$ - (x_0,t_0) nuqtadan $x \geq x_0$ o‘tuvchi $x+t=x_0+t_0$ va $x-t=x_0-t_0$ to‘g‘ri chiziqlardan tashkil topgan Oxt tekislikdagi soha bo‘lsin. $(x_0,t_0)=(0,0)$ bo‘lgan holni qaraymiz.[1]

Ushbu

$$\begin{cases} u_{tt} - u_{xx} + \gamma b(u_t\text{-}v_t) = f(x,t) \\ v_{tt} - b(u_t\text{-}v_t) = f(x,t) \end{cases}, \tag{1}$$

tenglamalar sistemasining $D = \{x,t : 0 \le x - t \le 2L, 0 \le x + t \le 2L\}$ kvadratda

$$\begin{cases} u\big|_{t=x} = \tilde{\varphi}(x), \quad 0 \le x \le L, \\ u\big|_{t=-x} = \tilde{\psi}(x), \quad 0 \le x \le L, \\ \tilde{\varphi}(0) = \tilde{\psi}(0), \\ v(x,0) = 0, \; v_t(x,0) = 0 \end{cases} \tag{2}$$

shartlarni qanoatlantiruvchi $u(x,t)$ va $v(x,t)$ yechimlari topilsin.

(1) sistemadagi $u(x,t)$ va $v(x,t)$ mos ravishda ρ_s va ρ_l o‘zgarmas parsial zichlikka ega g‘ovak-elastik jism va suyuqlik zarralarining siljish tezligi vektorining komponentalaridir. Soddalik uchun siljish moduli o‘zgarmas va ko‘ndalang to‘lqin tarqalish tezligini birga teng deb olamiz. $b -$ Darsi koeffitsiyenti, $L > 0 -$ berilgan son, $\gamma = \rho_l / \rho_s$.[2]

1-ta’rif. *Agar* $u,\ v,\ u_t,\ v_t,\ u_{tt},\ v_{tt},\ u_{xx} \in C(D)$ *va* $u(x,t),\ v(x,t)$ *(1) tenglamalar sistemasini va (2) shartlarni qanoatlantirsa,* $u(x,t)$ *va* $v(x,t)$ *funksiyalar (1), (2) masalaning yechimi deyiladi.*

Ushbu funksiya

$$u(x,\ t) = \tilde{f}(x-t) + \tilde{g}(x+t) + \frac{1}{2}\int_0^t \int_{x-t+\tau}^{x+t-\tau} f(\xi,\tau)\,d\xi\,d\tau \tag{3}$$

($\gamma = 0$ bo‘lgan holdagi (1) sistemaning birinchi tenglamasida) quyidagi bir jinsli bo‘lmagan to‘lqin tenglamasining umumiy yechimi bo‘ladi:[3,4]

$$u_{tt} - u_{xx} = f(x,t) \tag{4}$$

(3) formuladagi $\tilde{f}(x)$ va $\tilde{g}(x)$ funksiyalar argumentlari ikki marta uzluksiz differensiallanuvchi ixtiyoriy funksiyalar.

(3) dan birinchi va (2) dagi ikkinchi munosabatlarga ko‘ra

$$\tilde{\varphi}(x) = \tilde{f}(0) + \tilde{g}(2x) + \frac{1}{2}\int_0^x \int_\tau^{2x-\tau} f(\xi,\tau)d\xi d\tau,$$

$$\tilde{\psi}(x) = \tilde{f}(2x) + \tilde{g}(0) + \frac{1}{2}\int_0^{-x} \int_{2x+\tau}^{-\tau} f(\xi,\tau)d\xi d\tau$$

ga ega bo‘lamiz. Bundan $y = 2x$ deb

$$\tilde{g}(y) = \tilde{\varphi}(y/2) - \tilde{f}(0) - \frac{1}{2}\int_0^{y/2} \int_\tau^{y-\tau} f(\xi,\tau)d\xi d\tau,$$

$$\tilde{f}(y) = \tilde{\psi}(y/2) - \tilde{g}(0) - \frac{1}{2}\int_0^{-y/2} \int_{y+\tau}^{-\tau} f(\xi,\tau)d\xi d\tau,$$

$$\tilde{\varphi}(0) = \tilde{f}(0) + \tilde{g}(0)$$

ni hosil qilamiz.

U holda (3) formula[5]

$$u(x,t) = \tilde{\varphi}((x+t)/2) + \tilde{\psi}((x-t)/2) - \tilde{\varphi}(0) + \frac{1}{2}\int_0^t \int_{x-t+\tau}^{x+t-\tau} f(\xi,\tau)d\xi d\tau -$$

$$-\frac{1}{2}\int_0^{(x+t)/2} \int_\tau^{x+y-\tau} f(\xi,\tau)d\xi d\tau - \frac{1}{2}\int_0^{-(x-t)/2} \int_{x-t+\tau}^{-\tau} f(\xi,\tau)d\xi d\tau$$

ko‘rinishga keladi. $\tilde{\varphi}(x)$ va $\tilde{\psi}(x)$ funksiyalar $0 \le x \le L$ da aniqlangani uchun $u(x,t)$ yechim faqat D kvadratda aniqlanadi.

Tenglama umumiyroq ko‘rinishda bo‘lsa, murakkabroq bo‘ladi. Hatto chiziqli integro-differensial tenglama[6-7]

$$u_{tt} - u_{xx} + \gamma b u_t - \gamma b^2 \int_0^t u_\eta(x,\eta)\exp[-b(t-\eta)]d\eta = g(x, t)$$

uchun ham (2) tenglamalar bilan yechimning aniq analitik ko‘rinishini olishi mumkin emas va amaliy masalalarning yechimida taqribiy yechimlarni qurishda turli usullardan foydalanish kerak bo‘ladi. Oxirgi tenglama (1) ning birinchi tenglamasidan $v(x,t)$ funksiyaning o‘rniga[8]

$$v(x,t) = \int_0^t (t-\eta)[bu_\eta(x,\eta) + f(x,\eta)]\exp[-b(t-\eta)]d\eta$$

ifodani qo‘yish orqali olinadi, bunda

$$g(x,t)=f(x,t)+\gamma b\int_0^t f(x,\eta)\exp[-b(t-\eta)]d\eta$$

SONLI NATIJALAR VA TAHLILI.

Quyidagi masalani qaraymiz:

$$u_{tt}-u_{xx}+\ \gamma bu_t-\gamma b^2\int_0^t u_\eta(x,\eta)\exp[-b(t-\eta)]d\eta\ =\ g(x,\ t),\qquad (5)$$

$$u\big|_{t=x}=\tilde{\varphi}(x),\quad 0\le x\le L,\qquad (6)$$

$$u\big|_{t=-x}=\tilde{\psi}(x),\quad 0\le x\le L,\qquad (7)$$

$$\tilde{\varphi}(0)=\tilde{\psi}(0).\qquad (8)$$

(5) tenglamaning birinchi qo‘shiluvchidan boshqa barcha hadini o‘ng tomonga o‘tkazib,

$$F(x,t)=\gamma b^2\int_0^t u_\eta(x,\eta)\exp[-b(t-\eta)]d\eta\ \text{-}\gamma bu_t\text{+}\ g(x,\ t)$$

deb belgilab, hosil bo‘lgan tenglamani (4) tenglamaga o‘xshash deb qarash mumkin, uning yechimini esa

$$u(x,\ t)=\tilde{\varphi}((x+t)/2)+\tilde{\psi}((x-t)/2)-\tilde{\varphi}(0)+\frac{1}{2}\int_0^t\int_{x-t+\tau}^{x+t-\tau}F(\xi,\tau)d\xi d\tau-$$

$$-\frac{1}{2}\int_0^{(x+t)/2}\int_{\tau}^{x+y-\tau}F(\xi,\tau)d\xi d\tau-\frac{1}{2}\int_0^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}F(\xi,\tau)d\xi d\tau=$$

$$=\varphi(x,t)+\frac{1}{2}\int_0^t\int_{x-t+\tau}^{x+t-\tau}F(\xi,\tau)d\xi d\tau-$$

$$-\frac{1}{2}\int_0^{(x+t)/2}\int_{\tau}^{x+y-\tau}F(\xi,\tau)d\xi d\tau-\frac{1}{2}\int_0^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}F(\xi,\tau)d\xi d\tau,\qquad (9)$$

ko‘rinishda ifodalash mumkin, bunda

$$\varphi(x,t)=\tilde{\varphi}((x+t)/2)+\tilde{\psi}((x-t)/2)-\tilde{\varphi}(0).$$

Agar integro-differensial operator A ni

$$A[u]=\frac{1}{2}\int_{0}^{t}\int_{x-t+\tau}^{x+t-\tau}F(\xi,\tau)d\xi d\tau-$$

$$-\frac{1}{2}\int_{0}^{(x+t)/2}\int_{\tau}^{x+y-\tau}F(\xi,\tau)d\xi d\tau-\frac{1}{2}\int_{0}^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}F(\xi,\tau)d\xi d\tau$$

formula orqali kiritsak, (2.1.9) tenglamani

$$u=A[u]+\varphi \tag{10}$$

ko‘rinishda yozish mumkin.

(10) tenglama Volterra integro-differensial tenglamasi deyiladi. (10) tenglama yechimining taqribiy qurish usullaridan biri bu ketma-ket yaqinlashish usuli bo‘lib, har bir keyingi yaqinlashish

$$u_n=A[u_{n-1}]+\varphi,\qquad n=1,2,\ldots \tag{11}$$

formula bilan oldingisiga ko‘ra aniqlanadi. Bunda u_0 - berilgan.

$u_0(x,\ t)=0$ funksiyani nolinchi yaqinlashish sifatida tanlaylik. U holda (11) yaqinlashish sxemasi quyidagicha bo‘ladi:

$$u_1(x,\ t)=\varphi(x,t)+\frac{1}{2}\int_{0}^{t}\int_{x-t+\tau}^{x+t-\tau}g(\xi,\tau)d\xi d\tau-$$

$$-\frac{1}{2}\int_{0}^{(x+t)/2}\int_{\tau}^{x+t-\tau}g(\xi,\tau)d\xi d\tau-\frac{1}{2}\int_{0}^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}g(\xi,\tau)d\xi d\tau,$$

$$u_n(x,\ t)=u_1(x,\ t)-$$

$$-\frac{1}{2}\int_{0}^{t}\int_{x-t+\tau}^{x+t-\tau}\left(\gamma b^2\int_{0}^{\tau}u_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau-$$

$$-\frac{1}{2}\int_{0}^{(x+t)/2}\int_{\tau}^{x+t-\tau}\left(\gamma b^2\int_{0}^{\tau}u_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau-$$

$$-\frac{1}{2}\int_{0}^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}\left(\gamma b^2\int_{0}^{\tau}u_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau. \tag{12}$$

(12) formulada

$$u_{(n-1)\eta}(\xi,\eta)=\frac{\partial u_{n-1}(\xi,\eta)}{\partial\eta}.$$

(12) formuladan quyidagi munosabat kelib chiqadi:

$$u_{nt}(x,\ t)=u_{1t}(x,\ t)-$$

$$-\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau u_{(n-1)\eta}(x+t-\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(x+t-\tau,\tau)\right)d\tau+$$

$$+\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau u_{(n-1)\eta}(x-t+\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(x-t+\tau,\tau)\right)d\tau-$$

$$-\frac{1}{2}\int_0^{(x+t)/2}\left(\gamma b^2\int_0^\tau u_{(n-1)\eta}(x+t-\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(\xi,\tau)\right)d\tau+$$

$$+\frac{1}{2}\int_0^{(t-x)/2}\left(\gamma b^2\int_0^\tau u_{(n-1)\eta}(x-t+\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b u_{(n-1)\tau}(x-t+\tau,\tau)\right)d\tau \quad (13)$$

$\{u_n(x,\ t)\},\ \{u_{nt}(x,\ t)\}$ ketma-ketliklar tekis yaqinlashuvchi ekanligini ko‘rsatamiz. Ikki ketma-ketliklarning ayirmasining iteratsiyasini $w_n=u_{n+1}-u_n$ deb olamiz. (12), (13) formulalardan

$$w_n(x,\ t)==-\frac{1}{2}\int_0^t\int_{x-t+\tau}^{x+t-\tau}\left(\gamma b^2\int_0^\tau w_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b w_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau-$$

$$-\frac{1}{2}\int_0^{(x+t)/2}\int_{\tau}^{x+t-\tau}\left(\gamma b^2\int_0^\tau w_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b w_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau-$$

$$-\frac{1}{2}\int_0^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}\left(\gamma b^2\int_0^\tau w_{(n-1)\eta}(\xi,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b w_{(n-1)\tau}(\xi,\tau)\right)d\xi d\tau, \quad (14)$$

$$w_{nt}(x,\ t)=$$

$$-\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau w_{(n-1)\eta}(x+t-\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b w_{(n-1)\tau}(x+t-\tau,\tau)\right)d\tau+$$

$$+\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau w_{(n-1)\eta}(x-t+\tau,\eta)\exp[-b(\tau-\eta)]d\eta\ \text{-}\gamma b w_{(n-1)\tau}(x-t+\tau,\tau)\right)d\tau-$$

$$-\frac{1}{2}\int_0^{(x+t)/2}\left(\gamma b^2\int_0^{\tau} w_{(n-1)\eta}(x+t-\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta \text{ -}\gamma b w_{(n-1)\tau}(\xi,\tau)\right)d\tau +$$

$$+\frac{1}{2}\int_0^{(t-x)/2}\left(\gamma b^2\int_0^{\tau} w_{(n-1)\eta}(x-t+\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta \text{ -}\gamma b w_{(n-1)\tau}(x-t+\tau,\tau)\right)d\tau. \quad (15)$$

munosabatlarni topamiz.

Faraz qilaylik, D kvadratda

$$|w_0|\le M, \qquad |w_{0t}|\le M, \qquad (16)$$

tengsizlik o‘rinli bo‘lsin. $M>0$ - biror o‘zgarmas son. Ma’lumki, $\tilde{\varphi}(x)$, $\tilde{\psi}(x)$ va $g(x,\ t)$ berilganlar uchun muvofiq shartlarda (16) shart bajariladi.

(14) va (15) formulalardan mojarant bo‘lgan

$$|w_1|\le CM\frac{(x+t)^2}{2!}, \qquad |w_{1t}|\le CM(x+t),$$

baholar kelib chiqadi, $C>0, b$ va γ ga bog‘liq o‘zgarmas.

Induksiya bo‘yicha barcha $n\ge 2$ uchun quyidagi:

$$|w_n|\le CM\frac{(x+t)^{n+1}}{(n+1)!}, \ |w_{n_t}|\le CM\frac{(x+t)^n}{n!}$$

baholashni oson isbotlash mumkin. Oxirgi tengsizliklarda nuqtalar D kvadratning ichida yotganligidan

$$|w_n|\le CM\frac{(2L)^{n+1}}{(n+1)!}, \qquad |w_{nt}|\le CM\frac{(2L)^n}{n!} \qquad (17)$$

tengsizlik kelib chiqadi.

(17) tengsizliklarning o‘ng tomonida $\exp(2L)$ eksponentasi yoyilmasining proporsionallik ko‘paytuvchilari aniqligida umumiy hadi turibdi.

Demak,

$$u_n = u_0 + w_1 + \ldots + w_{n-1},$$

$$\frac{\partial u_n}{\partial t} = \frac{\partial u_0}{\partial t} + \frac{\partial w_1}{\partial t} + \ldots + \frac{\partial w_{n-1}}{\partial t}$$

funksiyalar ketma-ketligi limit funksiyaga tekis yaqinlashadi va uni $u(x,\ t)$, $U(x,\ t)$ orqali belgilaymiz:

$$u(x,\ t)=\lim_{n\to\infty}u_n(x,t),\quad U(x,\ t)=\lim_{n\to\infty}\frac{\partial u_n(x,t)}{\partial t}.$$

(12) va (13) formulalarda $n\to\infty$ da limitga o‘tamiz. Natijada

$$u(x,\ t)=u_1(x,\ t)-$$
$$-\frac{1}{2}\int_0^t\int_{x-t+\tau}^{x+t-\tau}\left(\gamma b^2\int_0^\tau U(\xi,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(\xi,\tau)\right)d\xi d\tau-$$
$$-\frac{1}{2}\int_0^{(x+t)/2}\int_\tau^{x+t-\tau}\left(\gamma b^2\int_0^\tau U(\xi,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(\xi,\tau)\right)d\xi d\tau-$$
$$-\frac{1}{2}\int_0^{-(x-t)/2}\int_{x-t+\tau}^{-\tau}\left(\gamma b^2\int_0^\tau U(\xi,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(\xi,\tau)\right)d\xi d\tau,$$
$$U(x,\ t)=u_{1t}(x,\ t)-$$
$$-\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau U(x+t-\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(x+t-\tau,\tau)\right)d\tau+$$
$$+\frac{1}{2}\int_0^t\left(\gamma b^2\int_0^\tau U(x-t+\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(x-t+\tau,\tau)\right)d\tau-$$
$$-\frac{1}{2}\int_0^{(x+t)/2}\left(\gamma b^2\int_0^\tau U(x+t-\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(\xi,\tau)\right)d\tau+$$
$$+\frac{1}{2}\int_0^{(t-x)/2}\left(\gamma b^2\int_0^\tau U(x-t+\tau,\eta)\exp\left[-b(\tau-\eta)\right]d\eta\ \text{-}\gamma bU(x-t+\tau,\tau)\right)d\tau \quad (18)$$

ga ega bo‘lamiz. Bundan esa $U=u_t$ va $u(x,\ t)$ funksiyalar (2.1.10) integro-differensial tenglamani qanoatlantirishi kelib chiqadi. (2.1.10) tenglamani t bo‘yicha differensiallasak $u(x,\ t)$ funksiya (5) integro-differensial tenglamani ham qanoatlantirishi kelib chiqadi. (6), (7) shartlarning bajarilishi (9), (12) formuladan va $\varphi(x,t)$ funksiya ko‘rinishidan kelib chiqadi.

Endi (5)-(7) masala yechimining yagonaligini isbotlaymiz. (5) - (7) masala ikkita $u_1(x,t)$ va $u_2(x,t)$ turli yechimlarga ega bo‘lsin. Ularning ayirmasi $W = u_1 - u_2$ ni qaraylik. Bu funksiya

$$W(x,\ t) = -\frac{1}{2}\int_0^t \int_{x-t+\tau}^{x+t-\tau} \left(\gamma b^2 \int_0^\tau W_\eta(\xi,\eta)\exp[-b(\tau-\eta)]d\eta \text{ -}\gamma b W_\tau(\xi,\tau) \right) d\xi d\tau -$$

$$-\frac{1}{2}\int_0^{(x+t)/2} \int_\tau^{x+t-\tau} \left(\gamma b^2 \int_0^\tau W_\eta(\xi,\eta)\exp[-b(\tau-\eta)]d\eta \text{ -}\gamma b W_\tau(\xi,\tau) \right) d\xi d\tau -$$

$$-\frac{1}{2}\int_0^{-(x-t)/2} \int_{x-t+\tau}^{-\tau} \left(\gamma b^2 \int_0^\tau W_\eta(\xi,\eta)\exp[-b(\tau-\eta)]d\eta \text{ -}\gamma b W_\tau(\xi,\tau) \right) d\xi d\tau$$

bo‘lgan bir jinsli Volterra integro-differensial tenglamani qanoatlantiradi. (16) baholashlarga asosan

$$|W| \le M_1, \qquad |W_t| \le M_1, \tag{19}$$

baholashning to‘g‘riligini ko‘rsatish oson, bunda $M_1 > 0$ biror o‘zgarmas. U holda $w_n(x,\ t)$ uchun olingan baholashga o‘xshash ixtiyoriy funksiya uchun

$$|W_n| \le CM_1 \frac{(2L)^{n+1}}{(n+1)!}. \tag{20}$$

baholashni olish mumkin. (20) formuladan $W(x,\ t) \equiv 0$ ekanligidan $u_1(x,t) \equiv u_2(x,t)$ ekani kelib chiqadi. (5) - (7) masala yechimning yagonaligi isbotlandi. Shunday qilib, quyidagi teorema isbotlandi.

Teorema. *$f(x,t)$ funksiya D da uzluksiz va $\tilde{\varphi}(x)$ va $\tilde{\psi}(x)$ funksiyalar $[0,L]$ da ikki marta uzluksiz differensiallanuvchi bo‘lsin. U holda (1), (2) masalaning yagona yechimi mavjud.*

XULOSA

Giperbolik tipdagi tenglamalar sistemasi uchun Gursa tipidagi to‘g‘ri masala o‘rganildi. Maqolada masalaning matematik modeli tahlil qilinib, yechimning mavjudligi, yagonaligi va berilgan boshlang‘ich hamda chegaraviy shartlarga bog‘liqligi ko‘rib chiqildi. Shuningdek, giperbolik tenglamalar sistemasining xossalari va ularning amaliy qo‘llanish imkoniyatlari tahlil qilindi.

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