

O‘NG TOMONI NOMA‘LUM TOR TEBRANISH TENGLAMASI UCHUN TO‘G‘RI TO‘RTBURCHAKLI SOHADA TESKARI MASALA

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Annotatsiya: Mazkur maqolada to‘g‘ri to‘rtburchakli sohada aniqlangan giperbolik tipdagi differensial tenglama uchun teskari manba masalasi tadqiq etiladi. Tenglamaning o‘ng tomonida vaqtga bog‘liq ma‘lum funksiya va fazoviy o‘zgaruvchiga bog‘liq noma‘lum funksiya ko‘paytmasi qatnashadi. Masala boshlang‘ich, va chegaraviy hamda yakuniy vaqt momentida berilgan qo‘shimcha shartlar yordamida aniqlanadi. Tadqiqotda noma‘lum manba funksiyasini aniqlash, masala yechimning mavjudligi va yagonaligi ko‘rib chiqiladi.

Kalit so‘zlar: giperbolik tenglama, to‘lqin tenglamasi, teskari manba masalasi, boshlang‘ich-chegaraviy masala.

KIRISH. Giperbolik tipdagi differensial tenglamalar uchun teskari masalalar matematik fizikaning muhim va dolzarb yo‘nalishlaridan biri hisoblanadi. Bunday masalalar ko‘pincha to‘lqin jarayonlarida noma‘lum manba yoki koeffitsiyentlarni aniqlash zarurati bilan bog‘liq bo‘lib, ular seysmologiya, akustika, elektromagnit jarayonlar va muhandislik masalalarida keng qo‘llaniladi.

Teskari masalalar dastlab V.G.Romanovning monografiyasida matematik fizika tenglamalari uchun teskari masalalarning umumiy nazariyasi, jumladan, giperbolik tipdagi tenglamalar uchun korrektilik masalalari chuqur tahlil qilingan [1]. Teskari manba masalalarini o‘rganish bo‘yicha esa fundamental natijalar Jin Cheng, Shuai Lu va M.Yamamoto ishlarida keltirilgan bo‘lib, ularda chegaraviy o‘lchovlar asosida to‘lqin tenglamasi uchun manba funksiyasini aniqlash masalasi ko‘rib chiqilgan [2]. O.Imanuvilov va M.Yamamoto tomonidan bajarilgan ishlarda to‘lqin tenglamasi uchun teskari manba masalalari chegaraviy o‘lchovlar yordamida o‘rganilib, yagonalik va turg‘unlik natijalari olingan [3]. Ushbu yo‘nalish A.Hasanov ishlarida yanada rivojlantirilib, giperbolik tenglamalar uchun turli sinfdagi teskari manba masalalari va ularning amaliy qo‘llanmalari ko‘rib chiqilgan [4].

Ma'lumki, teskari masalalarning aksariyati nokorrekt bo'lib, ularni o'rganishda maxsus matematik yondashuvlar talab etiladi. Bu borada A.N.Tixonov va V.Y.Arseninning klassik ishlari muhim ahamiyatga ega bo'lib, ularda nokorrekt masalalarni yechish nazariyasi asoslab berilgan [5].

So'nggi yillarda yurtimizda olimlari tomonidan ham teskari masalalar faol o'rganilmoqda. Jumladan, A.K.Urinov va M.S.Azizovlar tomonidan noma'lum o'ng tomonga ega bo'lgan to'rtinchi tartibli differensial tenglamalar uchun chegaraviy masalalarni tadqiq etilgan [6]. D.A.Tursunov va hammualliflari ishlarida xususiy hosilali differensial tenglamalar uchun teskari masalalarning korrektilik masalalari ko'rib chiqilgan [7]. R.R.Ashurov va R.T.Zunnunov ishlarida aralash tipdagi tenglamalar uchun kasr tartibli hosilaning darajasini aniqlashga oid teskari masala tadqiq etilgan bo'lib, bu ishlar giperbolik tenglamalar uchun teskari masalalar bilan mazmunan yaqin hisoblanadi [8]. Shuningdek, E.Azizbayov hamda D.K.Durdiev va T.R.Suyarov tomonidan giperbolik tipdagi tenglamalar uchun noma'lum koeffitsientlarni aniqlashga oid teskari masalalar o'rganilgan bo'lib, ularda noan'anaviy shartlar qo'llanilgan [9,10]. Shuningdek, [11] ishda ham tor tebranish tenglamasi uchun o'ng tomoni noma'lum funksiya aniqlash tadqiq etilgan.

Mazkur maqolada yuqoridagi ishlarning mantiqiy davomi sifatida vaqtga bog'liq ma'lum funksiya hosilasi va fazoviy o'zgaruvchiga bog'liq noma'lum manba funksiyasi ishtirok etgan giperbolik tenglama uchun teskari masala qaraladi. Masala boshlang'ich, chegaraviy va yakuniy vaqt shartlari yordamida aniqlanib, yechimning mavjudligi va yagonaligi masalalariga alohida e'tibor qaratiladi.

Asosiy qism. $\Omega = \{(x,t) : 0 < x < p, 0 < t < T\}$ to'g'ri to'rtburchakli sohada

$$Lu \equiv u_{tt} - a^2 u_{xx} = Q(t)f(x) \quad (1)$$

tenglamani qaraylik, bu yerda p va T - berilgan haqiqiy musbat sonlar, $Q(t)$ - berilgan ma'lum funksiya, $u = u(x,t)$ va $f(x)$ - lar esa noma'lum funksiyalar.

Masalaning qo‘yilishi. Teskari masala. $\bar{\Omega}$ sohada shunday $u(x,t)$ va $f(x)$ funksiyalar aniqlansinki, $u(x,t) \in C_{x,t}^{1,1}(\bar{\Omega}) \cap C_{x,t}^{2,2}(\Omega)$, $f(x) \in C(0,p) \cap L[0,T]$ bo‘lib, ular birgalikda (1) tenglamani va $u(x,t)$ esa ushbu

$$u(x,0) = \varphi_1(x), \quad 0 \leq x \leq p, \quad (2)$$

$$u_t(x,0) = \varphi_2(x), \quad 0 < x < p, \quad (3)$$

boshlang‘ich shartlarni hamda

$$u(0,t) = 0, \quad u(p,t) = 0, \quad 0 \leq t \leq T; \quad (4)$$

$$u_t(x,T) = \varphi_3(x), \quad 0 \leq x \leq p \quad (5)$$

chegaraviy shartlarni qanoatlantirsin, bu yerda $\varphi_i(x)$, $i=1,2,3$ - berilgan ma‘lum funksiyalar.

Masala yechimining yagonaligi. Quyidagi teorema o‘rinli:

1-teorema. Agar T, p sonlar va $Q(t) \in C^2[0,T]$, $Q'(t) \in L(0,T)$ $Q'(t)$ funksiya $[0,T]$ ishorasini o‘zgartirmaydigan bo‘lib u uchun shunday $\delta_0 = const > 0$ son mavjud bo‘lsinki,

$$\lambda_n^2 \int_0^T Q(\tau) \cos a\lambda_n(T-\tau) d\tau \geq \delta_0 \quad (6)$$

tengsizlik o‘rinli bo‘lsin, u holda (1)-masala bittadan ortiq yechimga ega bo‘lmaydi,

bu yerda $\delta_0 = \frac{\lambda_1}{a} \left[|Q(T)| (|\sin a\lambda_1 T| + 1) - |Q(0)| \right] \neq 0$ $\lambda_n = n\pi/p$, $n \in N$.

Isbot. Teskaridan faraz qilamiz. Qo‘yilgan masala $f_1(x)$, $f_2(x)$ va $u_1(x,t)$ $u_2(x,t)$ yechimlarga ega bo‘lsin. U holda, ularning farqini

$$f_1(x) - f_2(x) = f_0(x), \quad (7)$$

$$u_1(x,t) - u_2(x,t) = u(x,t) \quad (8)$$

deb olsak $u(x,t)$ va $f_0(x)$ funksiyalar birgalikda

$$u_{tt} - a^2 u_{xx} = Q(t) f_0(x), \quad (9)$$

tenglamani, $u(x, t)$ funksiya esa (2), (3), (4) va (5) shartlarga mos bir jinsli shartlarni qanoatlantiradi.

Ma'lumki,

$$X_n(x) = \sqrt{\frac{2}{p}} \sin \lambda_n x, \quad \lambda_n = \frac{n\pi}{p}, \quad n = 1, 2, \dots \quad (10)$$

sistema $L_2(0, p)$ fazoda to'la ortonormallangan sistemani tashkil qiladi.

Quyidagi funktsiyani qaraymiz:

$$q_n(t) = \int_0^p u(x, t) X_n(x) dx, \quad n = 1, 2, \dots \quad (11)$$

(11) funktsiyani

$$q_{n,\varepsilon}(t) = \int_{\varepsilon}^{p-\varepsilon} u(x, t) X_n(x) dx, \quad 0 < \varepsilon < p, \quad (\varepsilon, p - \varepsilon) = \emptyset \quad (12)$$

ko'rinishida yozib olamiz.

(12) tenglikni t bo'yicha ikki marta differensiallab, quyidagi tenglikni hosil qilamiz:

$$q''_{n,\varepsilon}(t) = \int_{\varepsilon}^{p-\varepsilon} u_{xx}(x, t) X_n(x) dx.$$

(9) tenglamadan foydalanib oxirgi tenglikni quyidagicha yozish mumkin:

$$q''_{n,\varepsilon}(t) = a^2 \int_{\varepsilon}^{p-\varepsilon} u_{xx}(x, t) X_n(x) dx + Q(t) \int_{\varepsilon}^{p-\varepsilon} f_0(x) X_n(x) dx. \quad (13)$$

(13) tenglikni o'ng tarafidagi birinchi integralni ikki marta bo'laklab, so'ngra undan $\varepsilon \rightarrow 0$ limitga o'tib, quyidagi ifodani hosil qilamiz:

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} q''_{n,\varepsilon}(t) &= a^2 \lim_{\varepsilon \rightarrow 0} [X_n(p - \varepsilon) u_x(p - \varepsilon, t) - X_n(\varepsilon) u_x(\varepsilon, t)] - \\ &\quad - a^2 \lim_{\varepsilon \rightarrow 0} [X'_n(p - \varepsilon) u(p - \varepsilon, t) - X'_n(\varepsilon) u(\varepsilon, t)] - \\ &\quad - a^2 \lambda_n^2 \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^{p-\varepsilon} u(x, t) X_n(x) dx + Q(t) \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^{p-\varepsilon} f_0(x) X_n(x) dx. \end{aligned}$$

Bu ifodadan (4) chegaraviy shartlarga mos bir jinsli chegaraviy shartlardan va $X_n(x)$ funksiyaning xossalarida foydalanib, quyidagi tenglikni olamiz:

$$q_n''(t) = -a^2 \lambda_n^2 \int_0^p u(x,t) X_n(x) dx + Q(t) \int_0^p f_0(x) X_n(x) dx, \quad n=1,2,\dots$$

Oxirgi tenglikdan (11) tenglikni e'tiborga olib, quyidagi oddiy differensial tenglamani hosil qilamiz:

$$q_n''(t) + a^2 \lambda_n^2 q_n(t) = Q(t) \int_0^p f_0(x) X_n(x) dx, \quad n=1,2,\dots \quad (14)$$

Bu tenglamaning umumiy yechimi quyidagi ko'rinishga ega:

$$q_n(t) = C_{1n} \cos a\lambda_n t + C_{2n} \sin a\lambda_n t + \frac{f_n}{a\lambda_n} \int_0^t Q(\tau) \sin a\lambda_n(t-\tau) d\tau, \quad n=1,2,\dots, \quad (15)$$

bu yerda C_{1n} va C_{2n} - ixtiyoriy o'zgarmas sonlar, $f_n = \int_0^p f_0(x) X_n(x) dx$

Endi C_{1n} , C_{2n} va f_n koeffitsiyentlarni aniqlaymiz. Buning uchun (15) umumiy yechimni (2), (3) boshlang'ich va (5) chegaraviy shartlarga mos bir jinsli shartlardan, (11) tenglikka asosan hosil qilingan

$$q_n(0) = 0, \quad q_n'(0) = 0, \quad q_n'(T) = 0 \quad (16)$$

shartlarga bo'ysindirramiz. Natijada (16) ning dastlabki shartidan $C_{1n} = 0$, $C_{2n} = 0$ ekanligi kelib chiqadi. (16) ning uchinchi shartiga asosan esa

$$f_n \int_0^T Q(\tau) \cos \lambda_n(T-\tau) d\tau = 0,$$

tenglikka ega bo'lamiz. Hosil bo'lgan bu tenglikdan $\lambda_n \neq 0$ bo'lganligi uchun

$$f_n \cdot \lambda_n^2 \int_0^T Q(\tau) \cos \lambda_n(T-\tau) d\tau = 0,$$

Tenglikni hosil qilamiz. Budan esa (6), shartga asosan $f_n = 0$ ekanligi kelib chiqadi.

Yuqoridagilarga ko'rindiki, $q_n(t) = 0, \quad \forall t \in [0, T]$. U holda, (11) tenglik quyidagi ko'rinishni oladi:

$$\int_0^p u(x,t) X_n(x) dx = 0, \quad n=1,2,\dots$$

Demak, $u(x,t)$ funksiya (10) to'la sistemadagi barcha funksiyalar bilan ortogonal ekan. Bu esa $u(x,t)$ funksiya Ω sohaning deyarli hamma joyida $u(x,t) = 0$ ekanligi kelib chiqadi. $u(x,t) \in C_{x,t}(\overline{\Omega})$ ekanligidan va (2)-(5) shartlarga mos bir jinsli shartlardan $u(x,t) \equiv 0 \quad (x,t) \in (\overline{\Omega})$ ekanligi kelib chiqadi. Ushbu mulohazalarni

$$f_n = \int_0^p f_0(x) X_n(x) dx = 0, \quad n=1,2,\dots$$

tenglik uchun ham takrorlab, $f_0(x) \equiv 0$ ekanligini topamiz.

Buni e'tiborga olsak, (7) va (8) tengliklardan $f_1(x) = f_2(x)$, $u_1(x,t) = u_2(x,t)$ tengliklar kelib chiqadi. Bu esa (1) masala yechimga ega bo'lsa, u yagona bo'lishini bildiradi. 1-teorema isbotlandi.

Izoh. (6) tengsizlik o'rinli bo'lishini ta'minlovchi T , p va $Q(t)$ lar to'plami bo'sh emas. T va p ni tanlab olish mumkin, bu yerda a – berilgan son.

$$\Delta = \lambda_n^2 \int_0^T Q(\tau) \cos a\lambda_n(T-\tau) d\tau = \lambda_n^2 \left[-\frac{Q(\tau)}{a\lambda_n} \sin a\lambda_n(T-\tau) \Big|_0^T + \int_0^T \frac{\sin a\lambda_n(T-\tau) Q'(\tau)}{a\lambda_n} d\tau \right] =$$

berilgan integralni bo'laklab quyidagi ko'rinishga ega bo'lamiz,

$$= \frac{\lambda_n Q(T)}{a} \sin a\lambda_n T + \frac{\lambda_n}{a} \int_0^T \sin a\lambda_n(T-\tau) Q'(\tau) d\tau$$

endi berilgan ifodadagi integralni hisoblashda o'rta qiymat haqidagi teoremdan foydalanamiz,

$$\begin{aligned} \Delta &= \frac{\lambda_n Q(T)}{a} \sin a\lambda_n T + \frac{\lambda_n}{a} \sin a\lambda_n(T-t_0) \int_0^T Q'(\tau) d\tau = \\ &= \frac{\lambda_n}{a} \left[Q(T) \sin a\lambda_n T - \sin a\lambda_n(T-t_0) (Q(0) - Q(T)) \right]. \end{aligned}$$

Endi Δ ni kuchsizlantiramiz.

$$\begin{aligned}
 |\Delta| &\geq \frac{\lambda_n}{a} \left[|Q(T) \sin a\lambda_n T| - |\sin a\lambda_n (T-t_0)(Q(0) - Q(T))| \right] \geq \\
 &\geq \frac{\lambda_1}{a} \left[|Q(T) \sin a\lambda_1 T| - |(Q(0) - Q(T)) \sin a\lambda_1 (T-t_0)| \right] \geq \\
 &= \frac{\lambda_1}{a} \left[|Q(T)| |\sin a\lambda_1 T| - |Q(0) - Q(T)| \right] = \frac{\lambda_1}{a} \left[|Q(T)| (|\sin a\lambda_1 T| + 1) - |Q(0)| \right] = \delta_0
 \end{aligned}$$

Masala yechimning mavjudligi. Masala yechimining mavjudligini ko'rsatish uchun o'zgaruvchilarni ajratish usulidan foydalanamiz.

2-teorema. Agar $\varphi_1(x), \varphi_3(x) \in C^3[0, p] \cap C^4(0, p)$,

$$\varphi_1^{(3)}(x), \varphi_3^{(4)}(x) \in L_2(0, p), \quad \varphi_1^{(2i)}(0) = \varphi_1^{(2i)}(p) = 0, \quad \varphi_3^{(2i)}(0) = \varphi_3^{(2i)}(p) = 0, \quad i = \overline{0, 2},$$

$$\varphi_2(x) \in C^2[0, p] \cap C^3(0, p), \quad \varphi_2^{(3)}(x) \in L_2(0, p), \quad \varphi_2^{(2i)}(0) = \varphi_2^{(2i)}(p) = 0, \quad i = \overline{0, 1}$$

shartlar bajarilsa, masalaning yechimi mavjud.

Isbot. Avval (1) tenglamaga mos bir jinsli tenglamaning (4) shartlarni qanoatlantiruvchi trivial bo'lmagan yechimlarini topamiz. Bunday yechimni quyidagi ko'rinishda qidiramiz.

$$u(x, t) = X(x)T(t). \tag{17}$$

(17) ifodani (1) tenglamaga mos bir jinsli tenglamaga qo'yib, o'zgaruvchilarni ajratib, quyidagi tenglikni olamiz:

$$\frac{T''(t)}{a^2 T(t)} = \frac{X''(x)}{X(x)}.$$

Oxirgi tenglikni o'ng tomoni faqat x o'zgaruvchiga, chap tomoni esa faqat t o'zgaruvchiga bog'liq, bundan ko'rinadiki, bu tenglik o'zgarmas songa teng. Uni $-\lambda^2$ o'zgarmas songa tenglab, $X(x)$ va $T(t)$ funksiyalarga nisbatan quyidagi oddiy differensial tenglamalarni hosil qilamiz:

$$X''(x) + \lambda^2 X(x) = 0, \quad 0 < x < p; \tag{18}$$

$$T''(t) + a^2 \lambda^2 T(t) = 0, \quad 0 < t < T. \tag{19}$$

(18) tenglamaning umumiy yechimi quyidagi ko'rinishda aniqlanadi.

$$X(x) = m_1 \cos \lambda x + m_2 \sin \lambda x, \tag{20}$$

bu yerda $m_j, j = \overline{1,2}$ - ixtiyoriy o'zgarmas sonlar.

(4) chegaraviy shartlardan (17) ga asosan quyidagi shartlar kelib chiqadi:

$$X(0) = X(p) = 0,$$

(20) yechimni oxirgi shartlarga bo'ysindirib, trivial bo'lmagan quyidagi yechimlarni topamiz:

$$X_n(x) = m_{2n} \sin \frac{n\pi}{p} x, \quad n = 1, 2, \dots, \quad (21)$$

bu yerda $m_{2n} = \text{const} \neq 0, n \in N$ - ixtiyoriy o'zgarmas sonlar.

(21) da $m_{2n} = \sqrt{2/p}, n \in N$ deb olsak, (10) ko'rinishidagi to'la ortonormal sistemaga ega bo'lamiz.

(19) tenglamaning $\lambda = \lambda_n$ ga mos trivial bo'lmagan yechimini $T_n(t)$ bilan belgilasak, (1) tenglamaga mos bir jinsli tenglamaning (4) shartlarni qanoatlantiruvchi sanoqli sondagi $T_n(t)X_n(x)$ ko'rinishidagi yechimlarga ega bo'lamiz.

Endi qo'yilgan masalani yechimini quyidagi qator ko'rinishida qidiramiz:

$$u(x, t) = \sum_{n=1}^{\infty} u_n(t) \cdot X_n(x). \quad (22)$$

(22) qatorni (1) tenglamaga qo'yib quyidagi tenglamani olamiz:

$$\sum_{n=1}^{\infty} [u_n''(t) + a^2 \lambda_n^2 u_n(t)] X_n(x) = Q(t) f(x). \quad (23)$$

(23) tenglamani o'ng tarafidagi $f(x)$ funksiyani $(0, p)$ intervalda $\{X_n(x)\}$ sistema bo'yicha Fure qatoriga yoyamiz:

$$f(x) = \sum_{n=1}^{\infty} f_n \cdot X_n(x), \quad (24)$$

bu yerda $f_n = \int_0^p f(x) X_n(x) dx, n = 1, 2, \dots$

(24) ni (23) tenglamaga qo'yib quyidagi differensial tenglamani hosil qilamiz:

$$u_n''(t) + a^2 \lambda_n^2 u_n(t) = f_n \cdot Q(t), \quad n = 1, 2, \dots \quad (25)$$

(25) tenglamaning umumiy yechimini o‘zgarmaslarni variatsialash usuli bilan topamiz:

$$u_n(t) = \varphi_{1n} \cos a\lambda_n t + \varphi_{2n} \sin a\lambda_n t + \frac{f_n}{a\lambda_n} \int_0^t Q(\tau) \sin a\lambda_n(t - \tau) d\tau, \quad n = 1, 2, \dots, \quad (26)$$

bu yerda φ_{1n} va φ_{2n} - ixtiyoriy o‘zgarmas sonlar.

(26) va (10) larni (22) qatorga qo‘yib, (1) tenglamaning yechimini hosil qilamiz:

$$u(x, t) = \sqrt{\frac{2}{p}} \sum_{n=1}^{+\infty} \left\{ \varphi_{1n} \cos a\lambda_n t + \varphi_{2n} \sin a\lambda_n t + \frac{f_n}{a\lambda_n} \int_0^t Q(\tau) \sin a\lambda_n(t - \tau) d\tau \right\} \sin \lambda_n x \quad (27)$$

(27) yechimni (2), (3) va (5) shartlarga bo‘ysundirib φ_{1n} , φ_{2n} va f_n koeffisientlarini topamiz.

$$\varphi_{1n} = \int_0^p \varphi_1(x) X_n(x) dx, \quad n = 1, 2, \dots, \quad (28)$$

$$\varphi_{2n} = \frac{1}{a\lambda_n} \int_0^p \varphi_2(x) X_n(x) dx, \quad n = 1, 2, \dots \quad (29)$$

$$\varphi_{3n} = \int_0^p \varphi_3(x) X_n(x) dx, \quad n = 1, 2, \dots, \quad (30)$$

bo‘lib, f_n koeffisient quyidagi tenglik bilan aniqlanadi:

$$f_n = \frac{\lambda_n^2 \left[\varphi_{3n} + \lambda_n \varphi_{1n} a \sin a\lambda_n T - \lambda_n \varphi_{2n} a \cos a\lambda_n T \right]}{\lambda_n^2 \int_0^T Q(\tau) \cos a\lambda_n(T - \tau) d\tau} \quad n = 1, 2, \dots \quad (31)$$

Shunday qilib, (27) qatorning koeffisientlari (28), (29), (30) va (31) tengliklar bilan aniqlanib, uning o‘zi va undan t bo‘yicha ikki marta, x bo‘yicha ikki marta hadlab hosila olishdan hosil bo‘lgan funksional qatorlar $\bar{\Omega}$ da tekis yaqinlashsa, u holda (27) qatorning yig‘indisi qo‘yilgan masalaning yechimi bo‘ladi.

Lemma. $\forall t \in [0, T]$ va $n = 1, 2, \dots$ bo‘lganda quyidagi baholar o‘rinli bo‘ladi:

$$|u_n(t)| \leq [|\varphi_{1n}| + |\varphi_{2n}|] + C_1 (a\lambda_n)^{-1} |\bar{f}_n| \|Q\|_{L(0,T)}, \quad (32)$$

$$|u'_n(t)| \leq a\lambda_n [|\varphi_{1n}| + |\varphi_{2n}|] + C_2 |\bar{f}_n| \|Q\|_{L(0,T)},$$

$$|u_n''(t)| \leq a^2 \lambda_n^2 [|\varphi_{1n}| + |\varphi_{2n}|] + a \lambda_n C_3 |\bar{f}_n| \|Q\|_{L(0,T)} + C_4 |\bar{f}_n| \|Q\|_{C(0,T)},$$

bu yerda $C_j, j = \overline{1,4}$ - musbat o'zgarimas sonlar, \bar{f}_n sifatida (31) formulaning surati nazarda tutilmoqda.

Isbot. (26) tenglik bilan aniqlanadigan $u_n(t)$ funksiyani baholaymiz. Avvalambor (26) ning uchinchi hadini baholaymiz. Bunda

$$\begin{aligned} (a \lambda_n)^{-1} |f_n| \left| \int_0^t Q(\tau) \sin a \lambda_n (t - \tau) d\tau \right| &\leq \frac{|\bar{f}_n|}{a \lambda_n \delta_0} \left| \int_0^t Q(\tau) d\tau \right| \leq \frac{|\bar{f}_n|}{a \lambda_n \delta_0} \int_0^t |Q(\tau)| d\tau = \\ &= C_1 (a \lambda_n)^{-1} |\bar{f}_n| \|Q\|_{L(0,T)}, \quad C_1 = \frac{1}{\delta_0} \end{aligned}$$

U holda (26) uchun

$$|u_n(t)| \leq [|\varphi_{1n}| + |\varphi_{2n}|] + C_1 (a \lambda_n)^{-1} |\bar{f}_n| \|Q\|_{L(0,T)}.$$

o'rinli bo'ladi. Yuqoridagi kabi, $u_n'(t)$ va $u_n''(t)$ larni ham baholashimiz mumkin.

Lemma isbotlandi.

Endi (27) qatorni va

$$\frac{\partial^2 u}{\partial x^2} = - \sum_{n=1}^{\infty} a^2 \lambda_n^2 u_n(t) \cdot X_n(x), \quad (33)$$

$$\frac{\partial^2 u}{\partial t^2} = - \sum_{n=1}^{\infty} a^2 \lambda_n^2 u_n(t) \cdot X_n(x) + Q(t) \sum_{n=1}^{\infty} |f_n| \cdot X_n(x), \quad (34)$$

qatorlarni tekis yaqinlashishga tekshiramiz. Avval (33) qatorni tekshiraylik. Buning uchun unga mos

$$\sum_{n=1}^{\infty} a^2 \lambda_n^2 |u_n(t)|. \quad (35)$$

mojarant qatorni tuzamiz. Lemmadagi (32) tenglikni e'tiborga olib va o'zgarimaslarni qoldirib yozsak, quyidagi qator hosil bo'ladi:

$$\sum_{n=1}^{\infty} [a^2 \lambda_n^2 |\varphi_{1n}| + a^2 \lambda_n^2 |\varphi_{2n}| + a^2 \lambda_n^2 \cdot |\bar{f}|].$$

Demak,

$$\sum_{n=1}^{\infty} \left[a^2 \lambda_n^2 |\varphi_{1n}| + a^2 \lambda_n^2 |\varphi_{2n}| \right], \quad (36)$$

$$\sum_{n=1}^{\infty} \frac{\lambda_n}{a} \cdot |\bar{f}_n| \quad (37)$$

Qatorlarning yaqinlashuvchiligini isbotlashimiz kerak. Buning uchun (28) ni uch marta, (29) ni esa ikki marta bo‘laklab, 2-teoremada $\varphi_1(x)$ va $\varphi_2(x)$ funksiyalarga qo‘yilgan shartlarni hisobga olsak,

$$\varphi_{1n} = \sqrt{\frac{2}{p}} \left(\frac{p}{\pi n} \right)^3 \int_0^p \varphi_1^{(3)}(x) \cos \frac{\pi n}{p} x dx,$$

$$\varphi_{2n} = -\sqrt{\frac{2}{p}} \left(\frac{p}{\pi n} \right)^3 \int_0^p \varphi_2^{(2)}(x) \sin \frac{\pi n}{p} x dx,$$

tengliklarga ega bo‘lamiz. Hosil bo‘lgan oxirgi tengliklarni

$$\varphi_{1n} = \left(\frac{p}{\pi} \right)^3 \frac{\varphi_{2n}^{(3)}}{n^3}, \quad \varphi_{2n} = -\left(\frac{p}{\pi} \right)^3 \frac{\varphi_{1n}^{(2)}}{n^3}, \quad (38)$$

ko‘rinishida yozib olamiz, bu yerda

$$\varphi_{2n}^{(3)} = \sqrt{\frac{2}{p}} \int_0^p \varphi_1^{(3)}(x) \cos \frac{n\pi}{p} x dx, \quad \varphi_{1n}^{(2)} = \sqrt{\frac{2}{p}} \int_0^p \varphi_2^{(2)}(x) \sin \frac{n\pi}{p} x dx.$$

(38) ni (36) ga qo‘yib, quyidagi natijaga kelamiz:

$$\sum_{n=1}^{\infty} a^2 \left[\frac{|\varphi_{1n}^{(2)}}{n} + \frac{|\varphi_{2n}^{(3)}}{n} \right]. \quad (39)$$

Bu qatorning yaqinlashuvchanligi

$$\frac{|\varphi_{1n}^{(2)}}{n} \leq \frac{1}{2} \left[\left(\varphi_{1n}^{(2)} \right)^2 + \frac{1}{n^2} \right], \quad \frac{|\varphi_{2n}^{(3)}}{n} \leq \frac{1}{2} \left[\left(\varphi_{2n}^{(3)} \right)^2 + \frac{1}{n^2} \right],$$

Elementar tengsizliklarga asosan kelib chuvchi

$$\sum_{n=1}^{\infty} \left[\left(\varphi_{1n}^{(2)} \right)^2 + \left(\varphi_{2n}^{(3)} \right)^2 \right], \quad \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

qatorlarning yaqinlashishidan kelib chiqadi.

Birinchi qatorning yaqinlashuvchanligi bo‘lakli uzluksiz bo‘lgan $\varphi_1^{(3)}(x)$, $\varphi_2^{(2)}(x)$ funksiyalar uchun Bessel tengsizligidan kelib chiqadi. Ikkinchi umumlashgan garmonik $\sum_{n=1}^{\infty} \frac{1}{n^\alpha}$ qator bo‘lib, $\alpha > 1$ bo‘lgani uchun yaqinlashuvchi bo‘ladi. Demak, (36) qator yaqinlashadi. Endi (37) qatorning yaqinlashuvchiligini ko‘rsatishda (31) tenglik suratini (37) ga qo‘yib, quyidagi majorant qatorni hosil qilamiz:

$$\sum_{n=1}^{\infty} \left[\frac{\lambda_n^3}{a} |\varphi_{3n}| + \lambda_n^3 |\varphi_{1n}| - \lambda_n^3 |\varphi_{2n}| \right]. \quad (40)$$

(40) qatorni yaqinlashuvchiligini isbotlash uchun (28) va (30) ni to‘rt marta (29) ni uch marta, bo‘laklab, 2-teoremada $\varphi_1(x)$, $\varphi_2(x)$ va $\varphi_3(x)$ funksiyalarga qo‘yilgan shartlarni hisobga olsak, (40) qator

$$\sum_{n=1}^{\infty} \left[\frac{|\varphi_{3n}^{(4)}|}{a} + |\varphi_{1n}^{(4)}| + |\varphi_{2n}^{(3)}| \right] \frac{1}{n} \quad (41)$$

ko‘rinishini oladi, bu yerda

$$\varphi_{1n}^{(4)} = \sqrt{\frac{2}{p}} \int_0^p \varphi_1^{(4)}(x) \sin \frac{n\pi}{p} x dx, \quad \varphi_{2n}^{(3)} = -\sqrt{\frac{2}{p}} \int_0^p \varphi_2^{(3)}(x) \cos \frac{n\pi}{p} x dx,$$

$$\varphi_{3n}^{(4)} = \sqrt{\frac{2}{p}} \int_0^p \varphi_3^{(4)}(x) \sin \frac{n\pi}{p} x dx.$$

(41) qatorning yaqinlashishini huddi (39) qatorning yaqinlashishi kabi ko‘rsatish mumkin. Demak, (37) qator yaqinlashadi. U holda, (35) mojarant qator tekis yaqinlashadi. Demak, $\overline{\Omega}$ sohada (33) va (27) qatorlar absolyut va tekis yaqinlashadi. (34) dagi ikkinchi qatorning absolyut va tekis yaqinlashishi (37) qatorning yaqinlashishidan kelib chiqadi. U holda, (34) qator ham $\overline{\Omega}$ da absolyut va tekis yaqinlashadi. Shunday qilib, agar 1 va 2 teoremlar shartlari bajarilsa, (27), (33) va (34) qatorlar absolyut va tekis yaqinlashadi. U holda bu qatorlar birgalikda (1) tenglamani qanoatlantiradi va (27) ning yig‘indisi (2)-(5) shartlarni qanoatlantiradi

ya'ni qo'yilgan masalaning yechimi bo'ladi, bunda $f(x)$ - (24) ko'rinishidagi funksiya bo'lib, f_n esa (31) tenglik bilan aniqlanadi. *Teorema isbotlandi.*

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