

BUZILADIGAN IKKINCHI TARTIBLI ODDIY DIFFERENSIAL TENGLAMA UCHUN BOSHLANG‘ICH MASALA

AZIZOV M.S., MAHMUDOVA M.T.

FarDU dotsenti, muzaffar.azizov.1988@gmail.com

FarDU talabasi, mmahmudova2303@gmail.com

Annotatsiya: Ushbu maqolada o‘zgaruvchilari ikkinchi tartibli differensial tenglama uchun boshlang‘ich masala tadqiq etiladi. Tadqiqot jarayonida tenglamani soddalashtirish va uni klassik ko‘rinishga keltirish va undan foydalanib yechimni qurish jarayoni olib boriladi. Xususan, maxsus almashtirishlar yordamida berilgan tenglamani Bessel tenglamasiga keltirish imkoniyati o‘rganiladi. Natijada tenglamaning umumiy yechimi maxsus funksiyalar orqali ifodalanadi hamda boshlang‘ich shartlar asosida xususiy yechim aniqlanadi. Olingan natijalarning nazariy va amaliy ahamiyati tahlil etiladi.

Kalit so‘zlar: ikkinchi tartibli differensial tenglama, Bessel funksiyasi, analitik yechim, maxsus almashtirish, boshlang‘ich shart.

Kirish. Differensial tenglamalar nazariyasi matematika fanining muhim bo‘limlaridan biri bo‘lib, ular yordamida turli fizik, texnik va tabiiy jarayonlar modellashtiriladi. Ayniqsa, ikkinchi tartibli differensial tenglamalar mexanika, tebranishlar nazariyasi va issiqlik almashinuvi kabi ko‘plab sohalarda keng qo‘llaniladi. Amaliy masalalarda ko‘pincha o‘zgaruvchan koeffitsiyentli differensial tenglamalar uchraydi. Bunday tenglamalarni to‘g‘ridan-to‘g‘ri yechish har doim ham oson emas. Shu sababli ularni maxsus almashtirishlar yordamida soddalashtirish yoki oldindan o‘rganilgan standart tenglamalarga keltirish muhim ahamiyatga ega. Shu bois quyidagi masalani o‘rganish nazariy jihatdan muhim bo‘lib, u murakkab differensial tenglamalarni yechish usullarini chuqurroq tushunishga yordam beradi.

Asosiy qism. Masala. *Quyidagi*

$$x^2 y'' + xy' + (\beta x^\alpha - \nu^2) y = 0, \quad (1)$$

buziladigan tenglama berilgan. Ushbu

$$\lim_{x \rightarrow 0} x^{-\frac{2\nu}{\alpha}} y(x) = k_1 \quad \lim_{x \rightarrow 0} x^{\frac{2\nu}{\alpha} + 1} y'(x) = k_2 \quad (2)$$

boshlang'ich shartlarni bajaradigan $y(x)$ funksiya topilsin, bu yerda $\nu > 0$, $\alpha > 0$ va k_1 va k_2 berilgan haqiqiy sonlar.

Masala yechimini mavjudligi. (1) tenglamaning yechimini

$$y(x) = z(t), \quad t = \frac{2\sqrt{\beta}}{\alpha} x^{\frac{\alpha}{2}} \quad (3)$$

ko'rinishda qidiramiz, bu yerda $z(t)$ - yangi noma'lum funksiya.

(3) ifodadan kerakli hosilalarni hisoblab (1) tenglamaga qo'yib ba'zi soddalashtirishlarni bajarsak

$$z'' + \frac{1}{t} z' + \left(1 - \frac{(2\nu/\alpha)^2}{t^2}\right) z = 0 \quad (4)$$

ko'rinishdagi Bessel tenglamasiga keladi. Ma'lumki (4) tenglamaning umumiy yechimi

$$z(t) = C_1 J_{2\nu/\alpha}(t) + C_2 Y_{2\nu/\alpha}(t) \quad (5)$$

ko'rinishda aniqlanadi va bu yerda $J_{2\nu/\alpha}(t)$, $Y_{2\nu/\alpha}(t)$ - birinchi tur Bessel funksiyalari, C_1 va C_2 - ixtiyoriy o'zgarmaslardir.

(5) funksiyadan (3) ga asosan avvalgi o'zgaruvchilarga qaytsak, (1) tenglamaning umumiy yechimi quyidagicha aniqlanadi:

$$y(x) = C_1 J_{2\nu/\alpha}\left(2\sqrt{\beta}x^{\frac{\alpha}{2}}/\alpha\right) + C_2 Y_{2\nu/\alpha}\left(2\sqrt{\beta}x^{\frac{\alpha}{2}}/\alpha\right). \quad (6)$$

C_1 va C_2 - ixtiyoriy o'zgarmaslarni (2) boshlang'ich shartlarga bo'ysundirib topamiz. (2) boshlang'ich shartlarning birinchisiga asosan

$$\lim_{x \rightarrow 0} x^\nu \left[C_1 \left[\sum_{n=0}^{\infty} \frac{(x/2)^{2n+\frac{2\nu}{\alpha}} (-1)^n}{\Gamma(n+1)\Gamma\left(n+1+\frac{2\nu}{\alpha}\right)} \right] + C_2 \left[\frac{J_{2\nu/\alpha}\left(\frac{2\sqrt{\beta}}{\alpha}x^{\frac{\alpha}{2}}\right) \cos \frac{2\nu}{\alpha} \pi - J_{-2\nu/\alpha}\left(\frac{2\sqrt{\beta}}{\alpha}x^{\frac{\alpha}{2}}\right)}{\sin \frac{2\nu}{\alpha} \pi} \right] \right] = k_1$$

$x \rightarrow 0$ da limitga o'tsak quyidagi natijaga ega bo'lamiz:

$$C_1 = k_1 \Gamma\left(1 + \frac{2\nu}{\alpha}\right) \left(\frac{\alpha}{\sqrt{\beta}}\right)^{2\nu/\alpha}.$$

(2) boshlang'ich shartlarning ikkinchisidan foydalanib C_2 ni aniqlaymiz.

$$\lim_{x \rightarrow 0} x^{\frac{2\nu}{\alpha}+1} y'(x) = x^{\frac{2\nu}{\alpha}+1} \left(C_1 J_{2\nu/\alpha} \left(2\sqrt{\beta} x^{\frac{\alpha}{2}} / \alpha \right) + C_2 Y_{2\nu/\alpha} \left(2\sqrt{\beta} x^{\frac{\alpha}{2}} / \alpha \right) \right)' = k_2,$$

$$C_2 = \frac{k_2 \pi}{\nu \Gamma(2\nu/\alpha)} \left(\frac{\alpha}{\sqrt{\beta}} \right)^{\frac{2\nu}{\alpha}}.$$

Demak, barcha hisob-kitoblarni yakunlab, berilgan differensial tenglamaning umumiy yechimi quyidagicha yoziladi:

$$y(x) = k_1 \Gamma \left(1 + \frac{2\nu}{\alpha} \right) \left(\frac{\alpha}{\sqrt{\beta}} \right)^{\frac{2\nu}{\alpha}} J_{\frac{2\nu}{\alpha}} \left(\frac{2\sqrt{\beta}}{\alpha} x^{\frac{\alpha}{2}} \right) + \frac{k_2 \pi}{\Gamma(2\nu/\alpha)} \left(\frac{\alpha}{\sqrt{\beta}} \right)^{\frac{2\nu}{\alpha}} Y_{\frac{2\nu}{\alpha}} \left(\frac{2\sqrt{\beta}}{\alpha} x^{\frac{\alpha}{2}} \right).$$

Endi (6) umumiy yechimda $\nu > 0$, $\alpha > 0$ ekanligini e'tiborga olib uni (2) boshlang'ich shartlarga bo'ysundirib qo'yilgan masalaning yechimini

$$y(x) = k_1 \Gamma \left(1 + \frac{2\nu}{\alpha} \right) \left(\frac{\alpha}{\sqrt{\beta}} \right)^{\frac{2\nu}{\alpha}} J_{\frac{2\nu}{\alpha}} \left(\frac{2\sqrt{\beta}}{\alpha} x^{\frac{\alpha}{2}} \right) + \frac{k_2 \pi}{\nu \Gamma(2\nu/\alpha)} \left(\frac{\alpha}{\sqrt{\beta}} \right)^{\frac{2\nu}{\alpha}} Y_{\frac{2\nu}{\alpha}} \left(\frac{2\sqrt{\beta}}{\alpha} x^{\frac{\alpha}{2}} \right)$$

ko'rinishda aniqlaymiz.

Masala yechimining yagonaligi.

Teorema. *Qo'yilgan masalaning yechimi yagonadir.*

Isbot. Faraz qilamiz qo'yilgan masala $y_1(x)$ va $y_2(x)$ yechimlarga ega bo'lsin.

U holda ularning ayirmasi

$$y_1(x) - y_2(x) = k(x) \tag{7}$$

bo'ladi.

$k(x)$ funksiya qo'yilgan masalaga mos bir jinsli

$$x^2 k(x)'' + x k(x)' + (\beta x^\alpha - \nu^2) k(x) = 0 \tag{8}$$

$$k(0) = 0, \lim_{x \rightarrow 0} \left(x^{\frac{2\nu}{\alpha}} k(x) \right)' = 0 \tag{9}$$

masalani qanoatlantiradi.

(8) tenglamaning umumiy yechimi (6) ko'rinishdagi kabi aniqlanadi ya'ni

$$k(x) = C_1 J_{\frac{2\nu}{\alpha}} \left(2\sqrt{\beta} x^{\frac{\alpha}{2}} / \alpha \right) + C_2 Y_{\frac{2\nu}{\alpha}} \left(2\sqrt{\beta} x^{\frac{\alpha}{2}} / \alpha \right) \quad (10)$$

(10) umumiy yechimni (9) bir jinsli shartlarga bo‘ysundirsak $C_1 = 0$, $C_2 = 0$ bo‘lishi kelib chiqadi bundan $k(x) = 0$. (7) ga asosan esa $y_1(x) = y_2(x)$. Teorema isbotlandi.

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