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ERDEYI–KOBER ALMASHTIRISH OPERATORI YORDAMIDA BESSEL OPERATORI QATNASHGAN ODDIY DIFFERENSIAL TENGLAMA UCHUN KOSHI MASALASINI YECHISH

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Annotatsiya. Ushbu maqolada Bessel differensial operatori qatnashgan ikkinchi tartibli oddiy differensial tenglama uchun qo‘yilgan Koshi masalasini yechishning operator-analitik usuli tadqiq qilingan. Bunda qaralayotgan singulyar tenglamani Erdeyi–Kober kasr tartibli integral almashtirish operatori yordamida ekvivalent ravishda mos klassik (singulyarliksiz) Koshi masalasiga keltirilgan. Maqolada Erdeyi–Kober operatorining xossalari, uning Bessel operatori bilan o‘zaro kompozitsiyasi batafsil bayon qilingan. Keltirilgan munosabatdan foydalanib, dastlabki masalaning yechimi oshkor ko‘rinishda topilgan. Olingan natijalar nafaqat nazariy ahamiyatga ega, balki amaliy masalalarda — issiqlik o‘tkazuvchanlik, tebranishlar nazariyasi, kvant mexanikasi va o‘qsimmetrik masalalarda uchraydigan singulyar differensial tenglamalarni yechishda ham keng qo‘llanilishi mumkin.

Kalit so‘zlar: Bessel operatori, Erdeyi–Kober operatori, kasr tartibli integral, singulyar differensial tenglama, Koshi masalasi, almashtirish operatori, gipergeometrik funksiya.

KIRISH

Singulyar koeffitsientli differensial operatorlar nazariyasi zamonaviy matematik fizika va amaliy matematikaning eng faol rivojlanayotgan sohalaridan biri hisoblanadi. Bunday operatorlar ichida alohida o‘rin egallaydigani — Bessel differensial operatori bo‘lib, u silindrik va sferik koordinatalardagi Laplas operatorining radial qismi sifatida tabiiy ravishda paydo bo‘ladi va shuning uchun issiqlik o‘tkazuvchanlik, tebranishlar tarqalishi, kvant mexanikasi hamda potensiallar nazariyasiga oid ko‘plab masalalarda muhim rol o‘ynaydi [1, 2].

Bessel differensial operatori deb

$$B_{\eta}^{(x)} = x^{-2\eta-1}\frac{d}{dx}x^{2\eta+1}\frac{d}{dx} = \frac{d^2}{dx^2} + \frac{2\eta+1}{x}\frac{d}{dx} \qquad (1)$$

ifoda tushuniladi. Ushbu operator $x = 0$ nuqtada singulyarlikga ega bo‘lib, klassik Koshi masalasining qo‘yilishi va yechilishida bir qator nazariy qiyinchiliklar yuzaga keladi. Aynan shu sababli, bunday singulyar tenglamalar uchun klassik usullarni to‘g‘ridan-to‘g‘ri qo‘llab bo‘lmaydi va *almashtirish (transmutatsiya) operatorlari* apparatidan foydalanish zarurati paydo bo‘ladi.

Almashtirish operatorlari nazariyasi A. Erdélyi, H.Kober, J.Delsarte, B.M.Levitan va keyinchalik I.A.Kipriyanov, V.V.Katraxov, S.M.Sitnik va boshqa olimlarning ishlarida sezilarli rivojlanish bosqichini bosib o‘tdi [3, 4]. Bu nazariyaning asosiy g‘oyasi shundan iboratki, biror murakkab (xususan, singulyar) differensial operatorni soddaroq (odatda — ikkinchi tartibli klassik differensiallash operatorining kvadratiga) ekvivalent ravishda akslantiruvchi chiziqli operator quriladi va shu orqali murakkab masalani sodda masalaga keltirib o‘rganiladi.

Maqolaning maqsadi — *Erdeyi–Kober kasr tartibli integral operatori* yordamida (1) ko‘rinishdagi Bessel operatori qatnashgan oddiy differensial tenglama uchun qo‘yilgan Koshi masalasini ekvivalent ravishda klassik Koshi masalasiga keltirish va yechimning yopiq ko‘rinishini topishdir. Bu yondashuv yechimning analitik xossalarini batafsil tahlil qilish va sonli usullar uchun samarali boshlang‘ich yaqinlashishlarni qurishga imkon beradi.

ASOSIY TA’RIFLAR VA YORDAMCHI TASDIQLAR

Riman–Liuvill ma’nosidagi kasr tartibli integral va differensial operatorlarning modifikatsiyalari va ularning umumlashmalari nazariya va amaliyotda keng qo‘llaniladi. Ushbu modifikatsiyalar, xususan, Erdeyi–Kober operatorlarini o‘z ichiga oladi.

A.Erdeyi va H.Kober o‘z ishlarida kasr tartibli integrallarning quyidagi

$$I_{\eta,\alpha}\varphi(x)=\frac{2x^{-2(\eta+\alpha)}}{\Gamma(\alpha)}\int_0^x (x^2-t^2)^{\alpha-1}t^{2\eta+1}\varphi(t)dt\,,\ \alpha>0\,,\ \eta\geq -1/2 \qquad (2)$$

$$K_{\eta,\alpha}\varphi(x)=\frac{2x^{2\eta}}{\Gamma(\alpha)}\int_x^\infty (t^2-x^2)^{\alpha-1}t^{1-2\alpha-2\eta}\varphi(t)dt\,,\ \alpha>0\ ,\ \eta\geq -1/2 \qquad (3)$$

modifikatsiyalari kiritilgan. Bu integral operatorlar integral tenglamalar va matematikaning boshqa sohalariga qo‘llashda juda foydali bo‘ldi. (2), (3) operatorlar va ularning umumlashmalari Erdeyi–Kober operatorlari deb nomlanadi.

(2) operatorga teskari bo‘lgan operator

$$I_{\eta,\alpha}^{-1}\varphi(x)=\frac{2x^{-2\eta}}{\Gamma(p-\alpha)}\left(\frac{1}{2x}\frac{d}{dx}\right)^{p}\int\limits_{0}^{x}(x^{2}-t^{2})^{p-\alpha-1}t^{2(\eta+\alpha)+1}\varphi(t)dt, \tag{4}$$

ko‘rinishga ega, bu yerda $p=[\alpha]+1$, $0<\alpha<p$ yoki $-p<\alpha<0$, $[\alpha]$ – α soning butun qismi.

A.Erdeyi ishlarida (2) va (3) operatorlarning (1) Bessel differensial operatori bilan kompozitsiyasi chuqur o‘rganilgan.

Erdeyi–Kober integral operatorlari almashtirish operatori xossasiga ega. Shuning uchun biz qisqacha almashtirish operatorining ta’rifi va uning qo‘llanilishiga to‘xtalib o‘tamiz.

1-ta’rif. Faraz qilaylik, (A,B) operatorlar jufti berilgan bo‘lsin. Agar

$$TA=BT \ (\text{yoki } AT=TB) \tag{5}$$

munosabat bajarilsa, T operator almashtirish operatori (AO, *transmutatsiya*) deb ataladi.

(5) tenglik qat’iy ta’rif bo‘lishi uchun, A, B va T operatorlar ta’sir qiladigan funksiyalar fazosi yoki to‘plamini aniqlash zarur. [3, 5] monografiyalar almashtirish operatorlari nazariyasi va uning qo‘llanilishiga bag‘ishlangan.

(2) va (3) Erdeyi–Kober operatorlari parametrlarning ma’lum qiymatlarida klassik Sonin operatori va Puasson operatorining umumlashmasi hisoblanadi [6]. Bu faktning to‘g‘riligi quyidagi teoremadan kelib chiqadi [7]:

1-teorema. *Agar* $\alpha>0$, $\eta\geq-1/2$, $f(x)\in C^{2}(0,b)$, $b>0$ *hamda* $x^{2\eta+1}f(x)$ *funksiya nol nuqta atrofida integrallanuvchi bo‘lib, ushbu*

$$\lim_{x\to 0}x^{2\eta+1}f'(x)=0$$

shart bajarilsa, u holda

$$B_{\eta+\alpha}^{(x)}I_{\eta,\alpha}f(x)=I_{\eta,\alpha}B_{\eta}^{(x)}f(x).$$

tenglik o‘rinli bo‘ladi.

Shuni ta’kidlash kerakki, [8] monografiyada Erdeyi–Kober almashtirish operatori yuqori tartibli singulyar koeffitsiyentli giperbolik tipdagi tenglamalar uchun Koshi va Gurs masalalarini yechishda qo‘llanilgan.

ASOSIY NATIJA VA KOSHI MASALASINING YECHIMI

Quyidagi Koshi masalasini qaraymiz:

$$B^{(x)}_{\alpha-\frac{1}{2}} y(x) + y(x) \equiv y''(x) + \frac{2\alpha}{x} y'(x) + y(x) = 0, \quad x > 0 \tag{6}$$

$$y(0) = A_0, \qquad y'(0) = A_1, \tag{7}$$

bu yerda $A_0, A_1 \in R$ – boshlang‘ich shartlar.

Masala yechimini

$$y(x) = I_{-\frac{1}{2},\alpha} g(x) \tag{8}$$

ko‘rinishida qidiramiz, bu yerda $I_{-1/2,\alpha}$ – (2) tenglik bilan aniqlangan Erdeyi-Kober operatori, $g(x)$ esa noma’lum funksiya.

(8) belgilashdan foydalansak, (6) tenglama quyidagicha bo‘ladi:

$$B_{\alpha-\frac{1}{2}} I_{-\frac{1}{2},\alpha} g(x) + I_{-\frac{1}{2},\alpha} g(x) = 0 \tag{9}$$

(9) ga 1-teoremani qo‘llab,

$$I_{-\frac{1}{2},\alpha} B_{-\frac{1}{2}} g(x) + I_{-\frac{1}{2},\alpha} g(x) = I_{-\frac{1}{2},\alpha} \left(g''(x) + g(x) \right) = 0$$

tenglikni hosil qilamiz. Hosil qilingan tenglikning har ikkala tomoniga

$$I^{-1}_{-1/2,\alpha} \varphi(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^x (x^2 - t^2)^{-\alpha} t^{2\alpha} \varphi(t) dt$$

teskari operatorni ta’sir ettirib va $\varphi(x) = 0$ da $I^{-1}_{-1/2,\alpha} \varphi(x) = 0$ ekanligini e’tiborga olib,

$$g''(x) + g(x) = 0 \tag{10}$$

tenglamaga ega bo‘lamiz.

(10) tenglama ikkinchi tartibli sodda differensial tenglama bo‘lib, uning umumiy yechimi

$$g(x) = C_1 \cos x + C_2 \sin x \tag{11}$$

funksiyadan iborat, bu yerda C_1, C_2 – ixtiyoriy o‘zgarmaslar.

(11) ni (8) ga qo‘yamiz:

$$y(x) = I_{-\frac{1}{2},\alpha} g(x) = I_{-\frac{1}{2},\alpha} (C_1 \cos x + C_2 \sin x) =$$

$$=\frac{2C_1x^{1-2\alpha}}{\Gamma(\alpha)}\int_0^x (x^2-t^2)^{\alpha-1}\cos t\,dt+\frac{2C_2x^{1-2\alpha}}{\Gamma(\alpha)}\int_0^x (x^2-t^2)^{\alpha-1}\sin t\,dt=$$

$$=\frac{2C_1x^{1-2\alpha}}{\Gamma(\alpha)}y_1(x)+\frac{2C_2x^{1-2\alpha}}{\Gamma(\alpha)}y_2(x), \tag{12}$$

bu yerda

$$y_1(x)=\int_0^x (x^2-t^2)^{\alpha-1}\cos t\,dt,$$

$$y_2(x)=\int_0^x (x^2-t^2)^{\alpha-1}\sin t\,dt.$$

Dastlab, $y_1(x)$ integralni hisoblaymiz. Buning uchun $\cos t$ funksiyaning

$$\cos t=\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{(2n)!}$$

cheksiz qatorga yoyilmasidan foydalanib, integralni quyidagicha yozib olamiz:

$$y_1(x)=\int_0^x (x^2-t^2)^{\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{(2n)!}dt.$$

Integral ostidagi qator argumentning istalgan qiymatida absolyut va tekis yaqinlashuvchi ekanligidan foydalanib, integral va qatorning o‘rinlarini almashtiramiz:

$$y_1(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\int_0^x (x^2-t^2)^{\alpha-1}t^{2n}dt.$$

Integrallash o‘zgaruvchisini $t=xs$ yordamida almashtirib,

$$y_1(x)=x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\int_0^1 (1-s^2)^{\alpha-1}s^{2n}\,ds$$

tenglikni hosil qilamiz. Agar s integrallash o‘zgaruvchisini $s=\sqrt{z}$ deb almashtirish bajarsak, so‘nggi tenglik

$$y_1(x)=\frac{1}{2}x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}\int_0^1 z^{n-\frac{1}{2}}(1-z)^{\alpha-1}\,dz$$

ko‘rinishga keladi. Bu tenglikdagi integral Eylerning birinchi tur integrali (beta-funksiya) ekanini e’tiborga olsak, uni quyidagi yozib olishimiz mumkin:

$$y_1(x) = \frac{1}{2} x^{2\alpha-1} \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} B\left(n+\frac{1}{2},\alpha\right).$$

Beta- va gamma-funksiyalar orasidagi bog‘lanish formulasidan foydalanib, ya’ni

$$B\left(n+\frac{1}{2},\alpha\right) = \frac{\Gamma\left(n+\frac{1}{2}\right)\Gamma(\alpha)}{\Gamma\left(n+\alpha+\frac{1}{2}\right)}$$

tenglikni e’tiborga olib,

$$y_1(x) = \frac{1}{2} x^{2\alpha-1} \Gamma(\alpha) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\left(n+\alpha+\frac{1}{2}\right)}$$

ekanligini topamiz.

$$\Gamma(z+n) = (z)_n \Gamma(z)$$

formulani $\Gamma\left(n+\frac{1}{2}\right)$ funksiyaga qo‘llab, ya’ni

$$\Gamma\left(\frac{1}{2}+n\right) = \left(\frac{1}{2}\right)_n \Gamma\left(\frac{1}{2}\right)$$

tengliklarni e’tiborga olib

$$y_1(x) = \frac{1}{2} x^{2\alpha-1} \Gamma(\alpha) \Gamma\left(\frac{1}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \frac{\left(\frac{1}{2}\right)_n}{\Gamma\left(n+\alpha+\frac{1}{2}\right)}$$

tenglikni hosil qilamiz. Ushbu

$$\left(\frac{1}{2}\right)_n = \frac{(2n)!}{2^{2n}\cdot n!}$$

formulani so‘nggi tenglikka qo‘llab,

$$y_1(x) = \frac{1}{2} x^{2\alpha-1} \Gamma(\alpha) \Gamma\left(\frac{1}{2}\right) \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}\cdot\Gamma\left(n+\alpha+\frac{1}{2}\right)} \frac{x^{2n}}{n!}$$

yoki

$$y_1(x)=\frac{1}{2}x^{2\alpha-1}\Gamma(\alpha)\Gamma\left(\frac{1}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n}{\Gamma\left(n+\alpha+\frac{1}{2}\right)}\frac{\left(\frac{x}{2}\right)^{2n}}{n!}$$

ekanligini topamiz. Bu tenglikning o‘ng tomoni surat va maxrajini $\left(\frac{x}{2}\right)^{\alpha-\frac{1}{2}}$ ga ko‘paytirib hamda $n!=\Gamma(n+1)$ tenglikni e’tiborga olib

$$y_1(x)=\frac{1}{2}\left(\frac{x}{2}\right)^{\frac{1}{2}-\alpha}x^{2\alpha-1}\Gamma(\alpha)\Gamma\left(\frac{1}{2}\right)\sum_{n=0}^{\infty}\frac{(-1)^n\left(\frac{x}{2}\right)^{2n+\alpha-\frac{1}{2}}}{\Gamma(n+1)\Gamma\left(n+\alpha+\frac{1}{2}\right)}$$

tenglikka ega bo‘lamiz. Hosil qilingan tenglikdagi qator birinchi turdagi $\left(\alpha-\frac{1}{2}\right)$ indeksli Bessel funksiyasi bo‘lib, uni $J_{\alpha-\frac{1}{2}}(x)$ kabi yozish mumkin. Buni e’tiborga olib va ayrim soddalashtirishlarni bajarib, $y_1(x)$ ning quyidagi ifodasiga ega bo‘lamiz:

$$y_1(x)=2^{\alpha-\frac{3}{2}}x^{\alpha-\frac{1}{2}}\Gamma(\alpha)\Gamma\left(\frac{1}{2}\right)J_{\alpha-\frac{1}{2}}(x). \tag{13}$$

Endi

$$y_2(x)=\int_0^x(x^2-t^2)^{\alpha-1}\sin t\,dt$$

integralni hisoblaymiz. Bu integralda $t=xs$ almashtirish bajarish natijasida quyidagi tenglikni hosil qilamiz:

$$y_2(x)=x^{2\alpha-1}\int_0^1(1-s^2)^{\alpha-1}\sin(xs)ds.$$

$\sin(xs)$ funksiyaning

$$\sin(xs)=\sum_{n=0}^{\infty}\frac{(-1)^n(xs)^{2n+1}}{(2n+1)!}$$

Teylor qatoriga yoyilmasidan foydalanib, $y_2(x)$ ni quyidagicha yozamiz:

$$y_2(x)=x^{2\alpha-1}\int_0^1(1-s^2)^{\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n\left(xs\right)^{2n+1}}{(2n+1)!}ds\,.$$

Integral ostidagi qator argumentning istalgan qiymatida absolyut va tekis yaqinlashuvchi ekanligidan foydalanib, integral va qatorning o‘rinlarini almashtiramiz:

$$y_2(x)=x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}\int_0^1(1-s^2)^{\alpha-1}s^{2n+1}ds\,.$$

Tenglikning o‘ng tomonidagi integralda $s=\sqrt{z}$ almashtirish bajarsak,

$$y_2(x)=\frac{1}{2}x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}\int_0^1(1-z)^{\alpha-1}z^n dz=\frac{1}{2}x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}B\left(n+1,\alpha\right)$$

tenglik kelib chiqadi. Beta va gamma-funksiyalar orasidagi bog‘lanishdan foydalanib, ya’ni

$$B\left(n+1,\alpha\right)=\frac{\Gamma\left(n+1\right)\Gamma\left(\alpha\right)}{\Gamma\left(n+\alpha+1\right)}$$

munosabatni e’tiborga olib, so‘nggi tenglikni quyidagicha yozamiz:

$$y_2(x)=\frac{1}{2}x^{2\alpha-1}\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}\frac{\Gamma\left(n+1\right)\Gamma\left(\alpha\right)}{\Gamma\left(n+\alpha+1\right)}\,.$$

Ushbu

$$\left(2n+1\right)!=\left(\frac{3}{2}\right)_n\cdot 2^{2n}\cdot n!,$$

$$\Gamma\left(n+1\right)=n!,\ \Gamma\left(n+\alpha+1\right)=\left(\alpha+1\right)_n\Gamma\left(\alpha+1\right)$$

formulalarni e’tiborga olsak va ba’zi sodda hisoblashlarni bajarsak,

$$y_2(x)=\frac{1}{2}\frac{\Gamma\left(\alpha\right)}{\Gamma\left(\alpha+1\right)}x^{2\alpha}\sum_{n=0}^{\infty}\frac{(-1)^n(1)_n}{\left(\frac{3}{2}\right)_n\cdot\left(\alpha+1\right)_n}\frac{\left(\frac{x}{2}\right)^{2n}}{n!}$$

yoki

$$y_2(x)=\frac{1}{2}\frac{\Gamma(\alpha)}{\Gamma(\alpha+1)}x^{2\alpha}\sum_{n=0}^{\infty}\frac{(1)_n}{\left(\frac{3}{2}\right)_n\cdot(\alpha+1)_n}\frac{\left(-\frac{x^2}{4}\right)^n}{n!}$$

tenglikka kelamiz. Agar ${}_1F_2(a;b_1,b_2;z)$ gipergeometrik funksiya yoyilmasini e’tiborga olsak,

$$y_2(x)=\frac{1}{2}\frac{\Gamma(\alpha)}{\Gamma(\alpha+1)}x^{2\alpha}\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right) \tag{14}$$

tenglik hosil bo‘ladi.

$y_1(x)$ va $y_2(x)$ ning (13) va (14) tengliklar bilan aniqlangan ifodalarini (12) ga qo‘yamiz:

$$y(x)=C_1\left(\frac{x}{2}\right)^{\frac{1}{2}-\alpha}\Gamma\left(\frac{1}{2}\right)J_{\alpha-\frac{1}{2}}(x)+\frac{C_2x}{\Gamma(\alpha+1)}\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right). \tag{15}$$

Agar ${}_0F_1(b;-z)=\Gamma(b)\,z^{\frac{1-b}{2}}J_{b-1}(2\sqrt{z})$ formulani e’tiborga olsak, (15) tenglik quyidagicha bo‘ladi:

$$y(x)=\frac{C_1\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\alpha+\frac{1}{2}\right)}\,{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)+\frac{C_2x}{\Gamma(\alpha+1)}\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right). \tag{16}$$

Shunday qilib, (16) tenglik bilan aniqlangan funksiya (6) tenglamaning umumiy yechimi ekan.

Endi {(6), (7)} Koshi masalasining yechimini topish uchun (16) umumiy yechim formulasini (7) shartlarga bo‘ysundiramiz.

$y(0)=A_0$ shartdan

$$C_1=\frac{A_0\cdot\Gamma\left(\alpha+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)} \tag{17}$$

ekanligini topamiz.

(7) ning ikkinchi shartidan foydalanish uchun (16) dan hosila olamiz. Buning uchun

$$y'(x)=\frac{C_1\Gamma\left(\frac{1}{2}\right)}{\Gamma\left(\alpha+\frac{1}{2}\right)}\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)+\frac{C_2}{\Gamma(\alpha+1)}\frac{d}{dx}\left[x\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right)\right] \quad (18)$$

tenglikdagi

$$\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)\ \text{va}\ \frac{d}{dx}\left[x\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right)\right]$$

hosilalarni hisoblaymiz.

$$\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)=\frac{d}{dx}\sum_{n=0}^{\infty}\frac{1}{\left(\alpha+1/2\right)_m}\cdot\frac{\left(-\frac{x^2}{4}\right)^n}{n!}=\frac{d}{dx}\sum_{n=0}^{\infty}\frac{(-1)^n}{4^n\left(\alpha+1/2\right)_n}\cdot\frac{x^{2n}}{n!}=$$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n\cdot 2n}{4^n\left(\alpha+1/2\right)_n}\cdot\frac{x^{2n-1}}{n!}=\sum_{n=1}^{\infty}\frac{(-1)^n\cdot 2n}{4^n\left(\alpha+1/2\right)_n}\cdot\frac{x^{2n-1}}{n!}.$$

$n=m+1$ almashtirish bajarsak,

$$\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)=-\frac{x}{2}\sum_{m=0}^{\infty}\frac{(-1)^m\cdot(m+1)}{4^m\left(\alpha+1/2\right)_{m+1}}\cdot\frac{x^{2m}}{(m+1)!}=$$

$$=-\frac{x}{2}\sum_{m=0}^{\infty}\frac{(-1)^m}{4^m\left(\alpha+1/2\right)_{m+1}}\cdot\frac{x^{2m}}{m!}=-\frac{x}{2}\sum_{m=0}^{\infty}\frac{(-1)^m}{\left(\alpha+1/2\right)_{m+1}}\cdot\frac{\left(-x^2/4\right)^m}{m!}.$$

$\left(\alpha+1/2\right)_{m+1}=\left(\alpha+1/2\right)\left(\alpha+3/2\right)_m$ ayniyatni e’tiborga olsak,

$$\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)=-\frac{x}{2\alpha+1}\sum_{m=0}^{\infty}\frac{1}{\left(\alpha+3/2\right)_m}\cdot\frac{\left(-x^2/4\right)^m}{m!},$$

$$\frac{d}{dx}{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)=-\frac{x}{2\alpha+1}\,{}_0F_1\left(\alpha+\frac{3}{2};-\frac{x^2}{4}\right). \quad (19)$$

$$\frac{d}{dx}\left[x\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right)\right]=\frac{d}{dx}\left[x\cdot\sum_{n=0}^{\infty}\frac{(1)_n}{(3/2)_n(\alpha+1)_n}\cdot\frac{\left(-x^2/4\right)^n}{n!}\right]=$$

$$=\frac{d}{dx}\left[\sum_{n=0}^{\infty}\frac{(-1)^n(1)_n}{4^n(3/2)_n(\alpha+1)_n}\cdot\frac{x^{2n+1}}{n!}\right]=\sum_{n=0}^{\infty}\frac{(-1)^n(1)_n(2n+1)}{4^n(3/2)_n(\alpha+1)_n}\cdot\frac{x^{2n}}{n!}.$$

$2n+1=\dfrac{(3/2)_n}{(1/2)_n}$ ayniyatni e’tiborga olsak,

$$\frac{d}{dx}\left[x\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right)\right]=\sum_{n=0}^{\infty}\frac{(1)_n}{(1/2)_n(\alpha+1)_n}\cdot\frac{\left(-x^2/4\right)^n}{n!},$$

$$\frac{d}{dx}\left[x\,{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right)\right]={}_1F_2\left(1;\frac{1}{2},\alpha+1;-\frac{x^2}{4}\right). \tag{20}$$

(19) va (20) ni (18) ga qo‘yamiz:

$$y'(x)=-\frac{C_1\Gamma(1/2)}{\Gamma(\alpha+1/2)}\frac{x}{2\alpha+1}\,{}_0F_1\left(\alpha+\frac{3}{2};-\frac{x^2}{4}\right)+\frac{C_2}{\Gamma(\alpha+1)}\,{}_1F_2\left(1;\frac{1}{2},\alpha+1;-\frac{x^2}{4}\right).$$

$y'(0)=A_1$ shartdan foydalanib,

$$C_2=A_1\cdot\Gamma(\alpha+1) \tag{21}$$

ekanligini topamiz.

C_1 va C_2 noma’lum o‘zgarmaslarning mos ravishda (17) va (21) tengliklar bilan aniqlangan ifodalarini (16) ga qo‘yamiz:

$$y(x)=A_0\cdot{}_0F_1\left(\alpha+\frac{1}{2};-\frac{x^2}{4}\right)+A_1x\cdot{}_1F_2\left(1;\frac{3}{2},\alpha+1;-\frac{x^2}{4}\right). \tag{22}$$

(22) formula bilan aniqlangan $y(x)$ funksiya {(6), (7)} Koshi masalasining yechimi bo‘ladi.

XULOSA

Ushbu maqolada Bessel singulyar differensial operatori qatnashgan ikkinchi tartibli oddiy differensial tenglama uchun qo‘yilgan Koshi masalasini Erdeyi–Kober kasr tartibli integral almashtirish operatori yordamida yechish usuli taklif etildi va asoslandi. Olingan natijalar issiqlik o‘tkazuvchanlik nazariyasi, tebranishlar tarqalish masalalari, kvant mexanikasi va boshqa amaliy sohalardagi singulyar differensial tenglamalarni yechishda qo‘llanilishi mumkin. Kelgusi tadqiqotlar yo‘nalishi sifatida masalani xususiy hosilali differensial tenglamalar va kasr tartibli singulyar tenglamalar uchun umumlashtirish mumkin.

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