COGNITIVE-NEUROSCIENTIFIC MECHANISMS OF ORAL ARITHMETIC DEVELOPMENT

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Abstract: The development of oral arithmetic competencies in primary school children constitutes a paradigm case for an interdisciplinary inquiry that integrates cognitive psychology, neuroscience, and philosophy of mind. The present paper proposes a theoretically grounded, integrative account of the cognitive-neuroscientific mechanisms underlying oral arithmetic development the Embodied Perceptual-Neurological Model (EPNM) and identifies three developmentally sensitive periods during which these mechanisms are most amenable to environmental input. Drawing on the 4E cognition framework (embodied, embedded, enactive, extended cognition), mechanistic explanation in cognitive neuroscience, and the neurodevelopmental theory of arithmetic, the model articulates how proto-arithmetical bodily resources, working-memory architectures, and culturally scaffolded strategies interact to produce oral arithmetic competence across ages 6–10. Particular attention is devoted to the philosophy-of-science dimensions of the inquiry: the status of mechanistic explanations in cognitive neuroscience, the theoretical challenges posed by the embodied turn, and the epistemological implications of sensitive period research for neuropedagogical design.

Keywords: oral arithmetic; embodied cognition; 4E cognition; cognitive neuroscience; mechanistic explanation; sensitive eriods; working memory; approximate number system; neuropedagogy; primary education.

1. Introduction: Oral Arithmetic as a Paradigm Case for 4E Cognition

Oral arithmetic – the capacity to perform exact numerical computation in the absence of external symbolic mediators – presents a uniquely instructive object of inquiry for the intersection of cognitive psychology, neuroscience, and philosophy of mind. Unlike written arithmetic, which distributes cognitive labour across agents and external representational systems, oral arithmetic constitutes a form of cognitive achievement that is, in a specific sense, more transparently "in the head" – and therefore seemingly in tension with the central claims of the 4E cognition programme (embodied, embedded, enactive, extended cognition). Yet a closer examination reveals that this tension is only apparent: oral arithmetic is deeply embodied in its developmental origins, culturally embedded in its strategic resources, and enactively constituted through the sensorimotor histories that shape its neural substrates.

The 4E cognition programme, represented in the pages of this journal by contributions from researchers such as Miłkowski (2016), Nowakowski (2020), and their colleagues, insists that cognitive processes cannot be adequately understood without reference to the body, the environment, and the active engagement of organisms with their niches. Mental arithmetic appears, superficially, to be a counter-example: it is rapid, internal, and apparently disembodied. The present paper argues, on the contrary, that the 4E framework provides the most theoretically productive perspective for understanding oral arithmetic development precisely because it reveals the embodied, culturally mediated, and enactive dimensions of a competence that is otherwise treated as a purely internal computational process.

The paper makes four principal contributions. First, it proposes the Embodied Perceptual-Neurological Model (EPNM) of oral arithmetic development – an integrative account that situates oral arithmetic within the theoretical framework of mechanistic explanation (Miłkowski, 2016) and embodied cognition. Second, it applies the framework of developmentally sensitive periods – windows of heightened neural plasticity most responsive to relevant environmental inputs – to the domain of oral arithmetic, identifying three such periods between ages 6 and 10. Third, it engages with the philosophy-of-science dimensions of the inquiry, examining the status of mechanistic explanations of arithmetic development within the broader debate between reductionist and integrative accounts. Fourth, it derives neuropedagogical design principles from the model.

The paper is organised as follows. Section 2 reviews the cognitive and neural foundations of arithmetic development through the lens of embodied cognition. Section 3 presents the EPNM and its four processing stages. Section 4 analyses the three sensitive developmental periods. Section 5 addresses philosophy-of-science implications and the embodied turn. Section 6 discusses neuropedagogical implications and design principles. Section 7 presents conclusions.

2. Embodied Cognitive and Neural Foundations of Arithmetic Development

2.1. The proto-arithmetical body: from finger counting to number sense

The embodied origins of arithmetical cognition are most conspicuously manifest in the practice of finger counting – a universally documented, cross-cultural practice that constitutes the primary bridge between the pre-symbolic numerical sense of infancy and the exact symbolic arithmetic of formal schooling. Embodied cognition research has extensively documented that finger counting is not merely a transitional scaffold but leaves lasting traces in the cognitive and neural architecture of adult arithmetic (Andres et al., 2007; Crollen & Noël, 2015). Adults solving simple arithmetic problems exhibit subthreshold motor activations in hand-motor areas – a neural echo of their embodied counting history. This finding instantiates the 4E cognition principle that bodily history constitutes cognitive architecture, rather than merely preceding it.

At the phylogenetic level, the capacity for numerical cognition is rooted in the Approximate Number System (ANS) – a biologically ancient, cross-species mechanism for representing numerical magnitude in analogue, non-symbolic format (Dehaene, 2011; Piazza, 2010). The ANS constitutes what Pantsar (2019) has characterised as a "proto-arithmetical" capacity: a pre-cultural, embodied numerical sense that provides the biological substrate upon which cultural practices of counting, symbolisation, and computation are superimposed through processes of enculturation. The neural substrate of the ANS – bilateral intraparietal sulcus (IPS) – is not a specialised "number module" that evolved for arithmetic, but a region whose quantitative sensitivity was originally related to spatial and magnitude processing more generally, illustrating the neural reuse principle (Miłkowski, 2016) that characterises much of higher cognitive architecture.

The transition from ANS-based proto-arithmetic to exact symbolic arithmetic – from approximate to precise, from embodied to culturally scaffolded – represents the central developmental challenge of the early primary school years. It is a transition that the embodied cognition framework characterises not as the replacement of a bodily resource by a cognitive-symbolic one, but as the progressive integration of bodily resources with cultural tools: symbolic numerals, counting words, arithmetic notations, and the classroom practices that deploy them.

Figure 1. Embodied foundations of arithmetic development: from proto-arithmetical body to cultural arithmetic

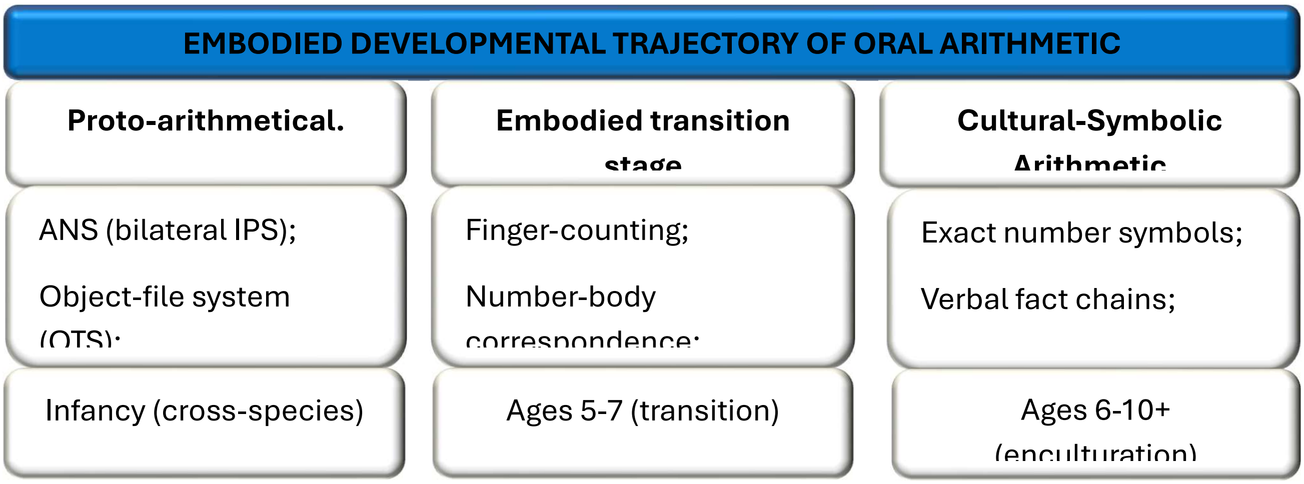

Figure 1. Embodied trajectory from proto-arithmetical biological resources to culturally scaffolded oral arithmetic (based on Pantsar, 2019; Dehaene, 2011; Andres et al., 2007)

2.2. The mechanistic architecture of arithmetic processing

The mechanistic programme in philosophy of science – represented in this journal by Miłkowski's (2016) analysis of cognitive neuroscience integration – provides the appropriate explanatory framework for understanding how the multiple components of oral arithmetic interact. A mechanistic explanation identifies the parts, operations, and organisations of a system that together produce a phenomenon (Craver, 2007). Applied to oral arithmetic, the phenomenon to be explained is rapid, accurate numerical computation performed in the absence of external support; the mechanism consists of the interacting components of ANS, working memory, strategy selection systems, and long-term fact memory, together with their neural substrates and organisational principles.

A central insight from the mechanistic framework, relevant for understanding oral arithmetic development, is the distinction between the mechanistic components and their organisation. Arithmetic development is not merely the addition of new components – more WM capacity, more ANS precision – but the progressive reorganisation of how components interact. This reorganisation is the developmental process by which proto-arithmetical bodily resources become integrated with cultural symbolic tools to produce culturally specific oral arithmetic competencies. It is precisely this organisational dimension that the embodied cognition framework – with its emphasis on body-environment-culture integration – is uniquely equipped to illuminate.

Working memory constitutes the central operational component of the arithmetic mechanism. Baddeley's (2012) multicomponent model distinguishes the phonological loop (verbal number word rehearsal), the visuospatial sketchpad (spatial numerical representation), the episodic buffer (integration), and the central executive (strategy selection and monitoring). Each component plays differential roles in oral arithmetic as a function of problem type, strategy, and developmental level. The phonological loop is preferentially engaged for sequential counting strategies; the visuospatial sketchpad for mental number line operations; the central executive for complex multi-step computation and strategy switching.

2.3. The enactive dimension: arithmetic as sense-making activity

The enactive tradition within 4E cognition (Varela et al., 1991; Thompson, 2007) characterises cognitive processes as forms of sense-making – activities through which an organism actively constitutes its cognitive environment rather than passively processing pre-given information. Applied to arithmetic, the enactive perspective highlights that children do not merely receive and compute numerical information but actively explore numerical regularities, construct strategies through action-perception loops, and constitute arithmetic meaning through their engagement with numerical environments.

This enactive dimension of arithmetic learning is most visible in the role of gesture in numerical cognition. Children's spontaneous gesture during arithmetic problem-solving constitutes an independent information channel that both reflects and supports their emerging understanding: gesture and speech can convey different, complementary information about a child's arithmetic knowledge, with gesture often prefiguring conceptual advances that have not yet been articulated verbally (Goldin-Meadow, 2003). This gesture-speech relationship exemplifies the enactive principle that cognitive processes are distributed across body and environment, not localised in internal mental representations alone.

3. The Embodied Perceptual-Neurological Model (EPNM) of Oral Arithmetic Development

3.1. Model overview and theoretical commitments

The Embodied Perceptual-Neurological Model (EPNM) integrates the mechanistic explanation framework, the 4E cognition programme, and the neurodevelopmental evidence reviewed in Section 2 into a four-stage developmental account of oral arithmetic competence. The model makes three explicit theoretical commitments that distinguish it from existing accounts. First, it adopts a mechanistic explanation strategy (Miłkowski, 2016; Craver, 2007) that identifies the components, operations, and organisations involved in oral arithmetic processing, rather than describing arithmetic development in purely behavioural or neural terms. Second, it incorporates the embodied cognition principle that proto-arithmetical bodily resources are constitutive components of the arithmetic mechanism, not merely developmental precursors to be transcended. Third, it adopts a developmental, sensitive-period perspective, recognising that the mechanistic organisation of oral arithmetic changes systematically over development in ways that create windows of differential educational responsiveness.

The EPNM posits four processing stages – Embodied Encoding, Distributed Working Memory Integration, Enactive Strategy Selection, and Automatised Fact Retrieval – that correspond to the dominant mode of arithmetic processing at successive developmental junctures. Unlike classical stage theories, these stages are not discrete or sequential for any given computation; in expert oral arithmetic, all four operate in rapid, partly parallel fashion. They are, rather, a developmental sequence: the predominant processing mode shifts progressively from Stage 1 toward Stage 4 as arithmetic competence develops.

Figure 2. The Embodied Perceptual-Neurological Model (EPNM): four stages and associated neural networks

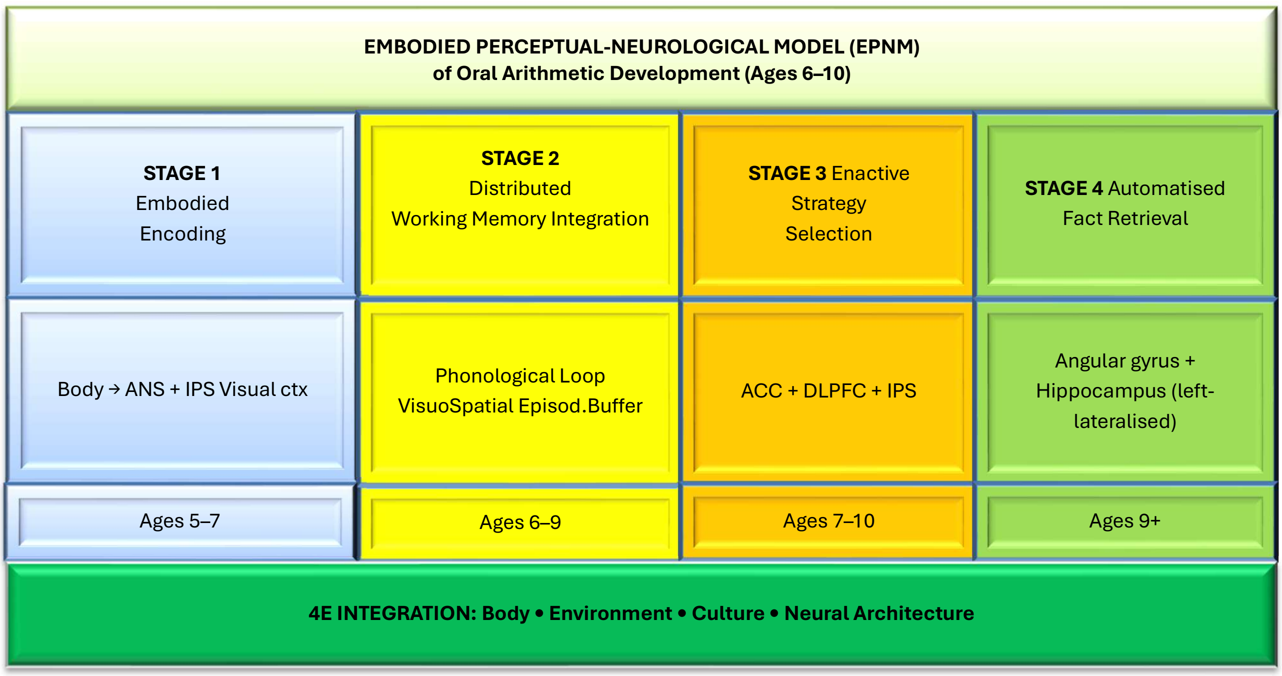

Figure 2. The EPNM: four developmental stages, dominant neural networks, and approximate age ranges (EPNM = Embodied Perceptual-Neurological Model; WM = working memory; ACC = anterior cingulate cortex; DLPFC = dorsolateral prefrontal cortex; IPS = intraparietal sulcus)

3.2. Stage 1: Embodied Encoding

Stage 1 encompasses the processes by which numerical problems are perceptually encoded and grounded in magnitude representations. Crucially, the EPNM characterises this as an embodied process in a technically precise sense: the encoding of numerical symbols activates motor resonances associated with the child's finger-counting history (Andres et al., 2007), visuospatial representations anchored in the spatial-numerical associations established through bodily counting practices, and ANS activations that provide an analogue magnitude framework for interpreting symbolic quantities. The phenomenon of "SNARC effect" (Spatial-Numerical Association of Response Codes) – whereby small numbers are faster to respond to with the left hand and large numbers with the right – is a marker of this embodied grounding of number representations in spatial-motor experience.

The encoding of number symbols in Stage 1 is therefore not a modality-neutral, amodal process but a body-relative one: what a number "is" for a developing child is partly constituted by its sensorimotor resonances and its position in the embodied number space. This insight, theorised in the 4E cognition literature as "conceptual grounding" or "embodied concept formation" (Varela et al., 1991), has direct pedagogical implications: Stage 1 instruction should not aim to abstract away from bodily number representations but to enrich and systematise them.

3.3. Stage 2: Distributed Working Memory Integration

Stage 2 involves the integration of encoded problem components within the distributed working memory system. The EPNM adopts Baddeley's (2012) multicomponent model but situates it within the wider cognitive ecology of the child: the phonological loop is engaged by the verbal number-word stream from the social environment; the visuospatial sketchpad by the spatial representations that embody numerical magnitude; the central executive by the metacognitive monitoring that is scaffolded by social interaction and explicit instruction. Working memory, on this account, is not a purely internal resource but an interface between the child's internal cognitive architecture and the distributed, socially scaffolded numerical environment.

The meta-analytic evidence confirms robust associations between working memory capacity and arithmetic performance across the primary school years (Peng et al., 2016; Raghubar et al., 2010). The EPNM interprets this association within the mechanistic framework: WM is a component in the arithmetic mechanism, and variations in WM capacity and efficiency systematically affect the mechanism's output. Critically, WM in the context of oral arithmetic is not merely a passive container for number representations but an active integrative process that participates in the constitution of arithmetic meaning – a point that aligns with the enactive characterisation of working memory as sensorimotor engagement rather than static storage (Nowakowski, 2020).

3.4. Stage 3: Enactive Strategy Selection

Stage 3 involves the selection among available arithmetic strategies and their execution. The EPNM adopts Siegler's (1996) overlapping waves framework – which characterises strategy development as continuous, probabilistic, and multiple – but enriches it with an enactive dimension. Strategy selection, on the enactive account, is not a purely internal, representation-driven process but a dynamical coupling between the child's internal cognitive resources, the problem-as-presented-in-the-environment, and the social norms and expectations of the arithmetic classroom.

The enactive character of strategy selection is illustrated by the documented sensitivity of strategy choices to presentation format, social context, and embodied affordances (Goldin-Meadow, 2003). A child solving 47 + 38 aloud, in conversation with a teacher, may deploy different strategies than the same child solving it silently, alone – not because the internal arithmetic mechanism is different but because the cognitive system is differently coupled with its environment. This coupling is not noise to be factored out but constitutive of the arithmetic process, as the enactive tradition insists.

Table 1. Components of the EPNM and their embodied characterisation within the 4E cognition framework

EPNM Stage	Dominant Neural Network	Embodied Dimension	4E Characterisation	Developmental Age
Stage 1: Embodied Encoding	Bilateral IPS + motor cortex + visual cortex	Finger-counting resonances; SNARC; spatial-numerical associations	Embodied: body constitutes number representations	5–7 years
Stage 2: WM Integration	DLPFC + IFG + superior parietal lobule	Phonol. loop (verbal number stream); visuospatial pad (spatial magnitudes)	Embedded: WM interfaces internal and external resources	6–9 years
Stage 3: Enactive Strategy	ACC + DLPFC + IPS	Strategy-environment coupling; gesture-speech coordination	Enactive: strategy as dynamical organism-environment coupling	7–10 years
Stage 4: Fact Retrieval	Left angular gyrus + hippocampus + temporal cortex	Consolidated verbal-motor sequences (number word chains)	Extended: cultural tools internalised as neural patterns	9+ years

3.5. Stage 4: Automatised Fact Retrieval

Stage 4 represents the developmental endpoint of the arithmetic mechanism for specific problem instances: the direct retrieval of consolidated problem-answer associations from long-term memory, bypassing the effortful operations of Stages 2 and 3. The EPNM characterises this stage not as the elimination of the embodied dimensions of arithmetic but as their internalisation: the verbal chains and spatial-motor regularities that constituted Stages 1–3 are progressively consolidated as neural patterns in the left angular gyrus, supramarginal gyrus, and hippocampus – a process that Dehaene (2011) characterises as the cortical "recycling" of language-related circuits for arithmetic.

The extended cognition dimension of Stage 4 is significant: the arithmetic facts stored in long-term memory are not purely internal constructions but the internalised residua of extended interactions with cultural tools – counting words, arithmetic notation, abacus training, classroom drill practices. The differences between individuals and cultures in arithmetic fact organisation (e.g., the greater multiplication table coverage of Asian adults) reflect the differential cultural scaffolding of Stage 4 consolidation, illustrating the extended cognition principle that cognitive architecture is partly constituted by the cultural tools with which organisms engage (Clark & Chalmers, 1998).

4. Sensitive Developmental Periods for Oral Arithmetic Competence: A Neuropedagogical Framework

4.1. The concept of sensitive periods in cognitive development

Sensitive periods – bounded windows of heightened neural plasticity during which relevant environmental inputs produce particularly strong and durable cognitive effects – are well established in the neuroscience of perceptual and linguistic development (Knudsen, 2004; Thomas & Johnson, 2008). The concept has been more cautiously applied to complex cognitive competencies such as arithmetic, partly because of the difficulty of differentiating sensitive period effects from general learning efficiency differences. Nevertheless, converging evidence from developmental neuroimaging, training studies, and cross-cultural research supports the conclusion that oral arithmetic development is characterised by at least three periods of differential environmental responsiveness during the primary school years.

From the perspective of the mechanistic explanation framework, sensitive periods can be understood as windows during which the organisation of the arithmetic mechanism – the relations among components, not merely the components themselves – is most responsive to modification. It is not primarily that children learn more per unit time during sensitive periods but that the effects of learning on the mechanism's organisation are more lasting and more generative of further development.

Table 2. Sensitive developmental periods within the EPNM: characteristics and educational implications

Sensitive Period	Age Range	EPNM Stage Target	Neural Plasticity Marker	Educational Priority	Key Embodied Resources
Period I: Symbol-Body Mapping	6–7 years	Stage 1 (Embodied Encoding)	IPS plasticity; finger-counting trace consolidation	Systematic concrete-to-symbolic bridging with bodily grounding	Finger patterns; number line gestures; manipulatives
Period II: Enactive Strategy Formation	7–9 years	Stage 3 (Enactive Strategy Selection)	Prefrontal-parietal myelination; ACC sensitivity to strategy competition	Strategy instruction and metacognitive awareness training	Gesture-based strategy representation; classroom dialogue
Period III: Cultural Fact Consolidation	9–10 years	Stage 4 (Automatised Retrieval)	Angular gyrus specialisation; hippocampal consolidation during sleep	Spaced practice; sleep quality; low-anxiety cultural context	Cultural tool internalisation; verbal chain consolidation

4.2. Period I: Symbol-Body Mapping (ages 6–7)

The first sensitive period encompasses the emergence of systematic mappings between symbolic number representations and the bodily, spatial, and ANS-based magnitude representations that constitute proto-arithmetic. During this period, the IPS undergoes rapid functional specialisation in response to experience with symbolic numbers: fMRI studies demonstrate that the magnitude-sensitivity of IPS responses – their capacity to distinguish numerically close from numerically distant symbolic quantities – increases substantially over the first year of formal schooling (Ansari, 2008). This specialisation is experience-dependent: children with richer exposure to structured numerical activities at school entry show faster IPS specialisation trajectories.

The embodied grounding of this period is critical for the EPNM: symbol-body mapping is not merely the association of visual symbols with abstract quantity concepts but the integration of symbolic representations into the bodily-spatial number space constituted through finger-counting, number-line, and spatial-numerical experiences. Pedagogically, this implies that instruction during

Period I should systematically exploit bodily resources – finger patterns, number line gestures, manipulative-based activities – as the primary medium through which symbols acquire arithmetical meaning, rather than treating bodily resources as transitional scaffolds to be discarded in favour of pure symbolism.

4.3. Period II: Enactive Strategy Formation (ages 7–9)

The second sensitive period is characterised by the rapid expansion and reorganisation of arithmetic strategy repertoires, driven by the joint development of working memory capacity (Gathercole et al., 2004) and the metacognitive capacities that enable adaptive strategy selection (Flavell, 1979). During this period, the prefrontal-parietal networks supporting strategy evaluation and selection undergo extensive myelination (Paus et al., 2001), making them increasingly efficient mediators of the competition between alternative arithmetic strategies.

From the enactive perspective, the central developmental achievement of Period II is not the acquisition of specific strategies but the development of the child's capacity to couple flexibly with different numerical problems as distinct affordance structures: to "see" a problem as calling for decomposition, or for approximation-and-correction, or for direct retrieval, and to engage with it accordingly. This capacity – which might be characterised as arithmetical "sense-making" in the enactive tradition – is precisely what metacognitive training aims to cultivate. Research demonstrates that metacognitive arithmetic instruction during Period II produces lasting improvements in adaptive strategy choice that generalise across problem types (Kramarski & Mevarech, 2003).

Figure 3. Temporal overlap of sensitive periods and corresponding neuropedagogical intervention priorities

	6 yr	**7 yr**	**8 yr**	**9 yr**	**10 yr**
Period I (Symbol–Body Mapping)	■	■			
Period II (Strategy)		■	■	■	
Period III (Consolidation)			■	■	■
Priority	**P1**	**P1+2**	**P2+3**	**P2+3**	**P3**

Figure 3. Schematic temporal structure of three sensitive periods for oral arithmetic and corresponding neuropedagogical intervention priorities (P = Period; Pri. = primary priority; ■ = sensitive period active)

4.4. Period III: Cultural Fact Consolidation (ages 9–10)

The third sensitive period corresponds to the progressive consolidation of arithmetic facts into the long-term memory networks of the left angular gyrus, supramarginal gyrus, and hippocampus. This consolidation is, in the extended cognition sense, a process of cultural internalisation: the verbal number-word chains, multiplication table recitations, and arithmetic drill sequences that constitute the cultural practices of primary mathematics education are progressively converted from extended cognitive scaffolds into internal neural patterns.

The quality of sleep emerges as a particularly significant environmental variable during Period III, given the established role of hippocampal slow-wave sleep in memory consolidation (Walker & Stickgold, 2006). Children who obtain sufficient high-quality sleep during the period of intensive arithmetic fact practice show superior fact retention and retrieval speed. The emotional context of arithmetic practice also shapes consolidation: mathematics anxiety activates amygdala responses that interfere with hippocampal encoding, impeding consolidation (Lyons & Beilock, 2012). These findings indicate that cultural conditions surrounding arithmetic instruction – sleep patterns, classroom emotional climate, evaluative practices – are not peripheral but constitutive determinants of the consolidation mechanism during Period III.

5. Philosophy-of-Science Dimensions: Mechanistic Explanation and the Embodied Turn

5.1. Mechanistic explanation and the EPNM

The mechanistic explanation framework, as analysed by Miłkowski (2016) in the context of cognitive (neuro)science integration, provides the epistemological backbone of the EPNM. A mechanistic explanation, in Craver's (2007) formulation, identifies the parts, operations, and organisations of a mechanism that together produce a phenomenon. The EPNM is explicitly mechanistic in this sense: it identifies the components (ANS, WM systems, strategy selection mechanisms, long-term fact memory), the operations they perform (magnitude encoding, working memory integration, strategy evaluation and execution, fact retrieval), and the organisation that produces rapid, accurate oral arithmetic computation.

However, the EPNM is a non-reductionist mechanistic account. It does not claim that oral arithmetic is "nothing but" neural computation in specific brain regions; rather, it claims that the mechanism spans body, brain, and cultural environment. The body components of the mechanism – finger-counting resonances, spatial-numerical body schema, gestural strategy representations – are as mechanistically constitutive as the neural components. This is consistent with the non-reductive mechanistic approach articulated by Miłkowski (2016): mechanisms can include extra-cranial, bodily, and environmental components as genuine parts of the mechanism, not merely as external influences on internal processes.

The integration challenge that Miłkowski (2016) identifies – the difficulty of achieving principled theoretical unification across the multiple levels and disciplines involved in understanding cognitive mechanisms – is acute in the present context. Oral arithmetic development involves findings from developmental psychology, cognitive neuroscience, comparative cognition, cross-cultural linguistics, and educational science. The EPNM attempts a partial integration by identifying the mechanistic roles played by findings from each of these disciplines within a single theoretical framework – not a reductive unification, but an organisational one.

5.2. The embodied turn and its implications for cognitive neuroscience

The embodied turn in cognitive science – the movement from "sandwich" models of cognition (perception-cognition-action) toward 4E accounts that situate cognition in body-environment dynamics – has significant implications for cognitive neuroscience that remain incompletely worked out, as noted by Nowakowski (2020) in the context of psychopathology and Miłkowski (2016) in the context of cognitive science integration. For arithmetic cognition, the embodied turn implies that neuroimaging studies cannot be interpreted as straightforwardly locating arithmetic "in the brain": the neural activations associated with arithmetic are components of a wider mechanism that includes bodily and environmental components.

This implication creates methodological tensions that the EPNM acknowledges without resolving. Neuroimaging studies typically control the environment, exclude bodily activity (participants lie still in scanner bores), and use artificial arithmetic tasks – conditions that systematically strip away the embodied and enactive dimensions of arithmetic that the EPNM highlights. The neuroscientific findings reviewed in Sections 2 and 3 must therefore be interpreted with caution: they document the neural components of an arithmetic mechanism that, in its natural form, is more extended, bodily, and environmentally embedded than scanner conditions allow.

Table 3. Philosophy-of-science positioning of the EPNM: comparison with alternative approaches

Approach	Explanatory Strategy	Treatment of Body	Treatment of Culture	Relation to EPNM
Classical cognitive science	Functional-computational	Peripheral (input-output)	External influence	Partial convergence at Stages 3–4; extended by EPNM
Modular nativism (Chomsky/Fodor)	Domain-specific modules	Irrelevant to modules	Surface variation on innate structure	Rejected: arithmetic is not modular
Reductive neuroscience	Neural substrates only	Absent from explanation	Absent or behavioural	Partial: EPNM uses neuroscience but situates it within wider mechanism
4E cognition (Varela, Noë, Clark)	Body-environment-agent dynamics	Constitutive of cognition	Constitutive cognitive scaffold	Primary theoretical commitment of EPNM
Mechanistic explanation (Miłkowski)	Parts-operations-organisation of mechanisms	Bodily components of mechanisms	Cultural components of mechanisms	Epistemological framework of EPNM

6. Neuropedagogical Design Principles for Oral Arithmetic Instruction

6.1. From EPNM to neuropedagogy: translation principles

The translation of the EPNM into pedagogical recommendations requires fidelity to the theoretical commitments of the model while acknowledging the substantial distance between neuroscientific evidence and educational practice – a distance that Howard-Jones (2014) characterises as a "translation gap" and that Sigman et al. (2014) propose to bridge through genuinely bidirectional neuroscience-education dialogue. The EPNM-derived principles offered below are theoretical design principles that specify the mechanisms by which educational practices should operate, rather than prescriptive instructional protocols.

The EPNM generates six design principles for oral arithmetic instruction. These principles are derived from the model's theoretical commitments and the sensitive period analysis of Section 4, and are grounded in the empirical evidence reviewed in Sections 2 and 3. They constitute an internally coherent neuropedagogical framework that reflects the 4E cognition commitments of the EPNM throughout.

6.2. Six neuropedagogical design principles

Principle 1: Embody the number line (Period I priority). Oral arithmetic instruction during ages 6–7 should systematically and progressively establish the mental number line as a bodily-spatial resource. This means exploiting children's proprioceptive and gestural resources – walking number lines, finger-trace movements, spatial positioning games – to ground symbolic number representations in embodied spatial experience before and alongside the introduction of purely symbolic operations. The goal is to ensure that symbolic numbers are not merely associatively linked to abstract magnitudes but are genuinely embodied in spatial-motor experience.

Principle 2: Exploit gesture-speech coordination (Periods I-II). Children's spontaneous gesture during arithmetic – number-representing hand shapes, counting movements, directional gestures – constitutes a cognitive resource rather than an irrelevant motor by-product. Instruction should actively incorporate and respond to children's arithmetic gestures, using them as windows into emerging understanding and as scaffolds for strategy development. This principle is grounded in research on gesture as a medium of mathematical cognition (Goldin-Meadow, 2003).

Principle 3: Cultivate enactive strategy flexibility (Period II priority). Instruction during ages 7–9 should aim not to teach a single "correct" mental arithmetic method but to develop a flexible repertoire of strategies and the metacognitive capacity to select adaptively among them. This requires explicit modelling of multiple strategies, comparative analysis of strategy efficiency for different problem types, and the creation of classroom conditions in which children can explore, discover, and share strategic approaches – an enactive, participatory pedagogy rather than transmission-based instruction.

Principle 4: Orchestrate cultural scaffolds for consolidation (Period III priority). Arithmetic fact consolidation during ages 9–10 depends not only on practice frequency but on the quality of the cultural scaffolds within which practice occurs. These scaffolds include: spaced-repetition practice schedules that align with hippocampal consolidation dynamics; classroom routines that create low-anxiety conditions for arithmetic practice; and attention to the sleep contexts within which consolidation primarily occurs. The extended cognition principle – that cultural practices are partly constitutive of cognitive architecture – implies that these cultural conditions are educational variables of the first order.

Principle 5: Manage working memory load developmentally (all periods). Because working memory is the primary cognitive bottleneck in oral arithmetic across all developmental stages, instructional design should minimise extraneous cognitive load (Sweller, 2011) while ensuring that germane load – the cognitive effort productively invested in the construction of arithmetic schemas – is maintained at levels appropriate to the child's WM capacity. This principle implies developmental progression in task complexity that tracks WM development, scaffolded initially by external supports that are progressively withdrawn.

Principle 6: Sustain emotional safety and positive coupling with numerical environments (all periods). The amygdala-hippocampal interactions that underlie mathematics anxiety (Lyons & Beilock, 2012) represent a systemic interference with the arithmetic mechanism at multiple levels: reduced WM capacity under anxiety (via prefrontal-amygdala competition), disrupted consolidation during Period III (via hippocampal interference), and diminished enactive engagement with numerical environments (via avoidance). Maintaining emotional safety in arithmetic learning is therefore not an affective luxury but a mechanistic prerequisite for the efficient operation of the arithmetic development mechanism.

Table 4. Summary of EPNM-derived neuropedagogical design principles with theoretical grounding

Principle	Sensitive Period	EPNM Stage	4E Dimension	Mechanistic Rationale
1. Embody the number line	Period I (6–7)	Stage 1	Embodied	Symbol-body mapping requires spatial-motor grounding of number representations in IPS+motor cortex
2. Exploit gesture-speech	Periods I-II (6–9)	Stages 1-3	Enactive	Gesture constitutes a mechanistic component of arithmetic; coordinates internal and external resources

3. Enactive strategy flexibility	Period II (7–9)	Stage 3	Enactive	Strategy repertoire and adaptive selection develop through organism-environment coupling during ACC maturation
4. Cultural scaffolds for consolidation	Period III (9–10)	Stage 4	Extended	Cultural practices constitute the conditions for hippocampal-neocortical consolidation of arithmetic facts
5. WM load management	All periods	All stages	Embedded	WM as embedded cognitive interface requires load management calibrated to developmental WM capacity
6. Emotional safety	All periods	All stages	Embodied + Embedded	Amygdala-hippocampal interference disrupts arithmetic mechanism; safety is a mechanistic prerequisite

Figure 4. Integration of EPNM stages, sensitive periods, and neuropedagogical principles

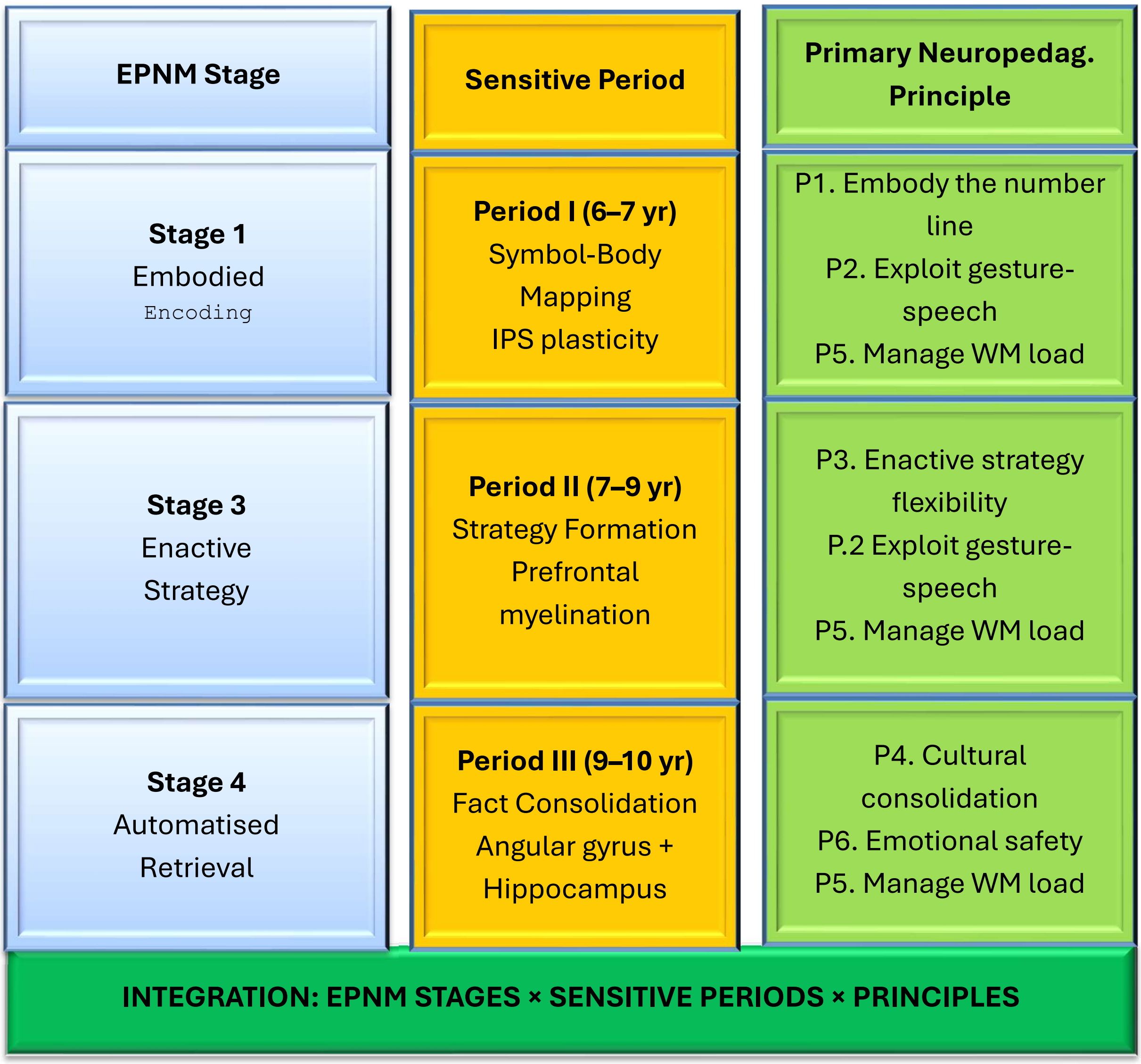

Figure 4. Summary integration matrix: EPNM stages, sensitive developmental periods, and primary neuropedagogical design principles

Figure 5. The EPNM within the landscape of 4E cognition approaches: conceptual map

CONCEPTUAL MAP: EPNM IN 4E COGNITION LANDSCAPE	
4E DIMENSION	EPNM APPLICATION TO ORAL ARITHMETIC
EMBODIED	Finger-counting resonances; SNARC; bodily number space; Stage 1 motor activations in arithmetic
EMBEDDED	WM as interface with numerical environment; classroom social scaffolding of strategy; culture
ENACTIVE	Arithmetic as sense-making; strategy as organism-environment coupling; gesture-speech coordination
EXTENDED	Cultural tools constituting neural architecture; internalised verbal chains; sleep consolidation

Figure 5. The EPNM as an integrative application of all four dimensions of 4E cognition to the domain of oral arithmetic development (SNARC = Spatial-Numerical Association of Response Codes; WM = working memory)

7. Conclusions

This paper has presented the Embodied Perceptual-Neurological Model (EPNM) of oral arithmetic development – a theoretically integrative account that situates oral arithmetic within the 4E cognition framework and the mechanistic explanation tradition represented in this journal. The EPNM proposes that oral arithmetic is a genuinely embodied, environmentally embedded, enactively constituted, and culturally extended cognitive achievement – not a counter-example to 4E cognition but one of its most instructive instantiations.

The model's four processing stages – Embodied Encoding, Distributed Working Memory Integration, Enactive Strategy Selection, and Automatised Fact Retrieval – correspond to a developmental progression from body-grounded proto-arithmetic to culturally internalised fact retrieval. Three overlapping sensitive developmental periods – Symbol-Body Mapping (ages 6–7), Enactive Strategy Formation (ages 7–9), and Cultural Fact Consolidation (ages 9–10) – characterise windows of differential neural plasticity and educational responsiveness within this progression.

Six neuropedagogical design principles derived from the EPNM – embodying the number line, exploiting gesture-speech coordination, cultivating enactive strategy flexibility, orchestrating cultural scaffolds for consolidation, managing WM load developmentally, and sustaining emotional safety – constitute a theoretically coherent framework for oral arithmetic instruction that reflects the 4E commitments of the model.

The paper has also engaged with the philosophy-of-science dimensions of the inquiry, arguing that a non-reductive mechanistic explanation strategy – one that treats bodily, environmental, and cultural components as genuine parts of the arithmetic mechanism – is both theoretically warranted and educationally productive. Future research should address the empirical validation of the EPNM's sensitive period predictions through longitudinal neuroimaging and cross-cultural training studies, the translation of the neuropedagogical principles into evaluated instructional programmers, and the refinement of the embodied cognition account of arithmetic development in dialogue with empirical advances in cognitive neuroscience and the philosophy of 4E cognition.

Figure 6. The EPNM: convergent evidence base and theoretical integration

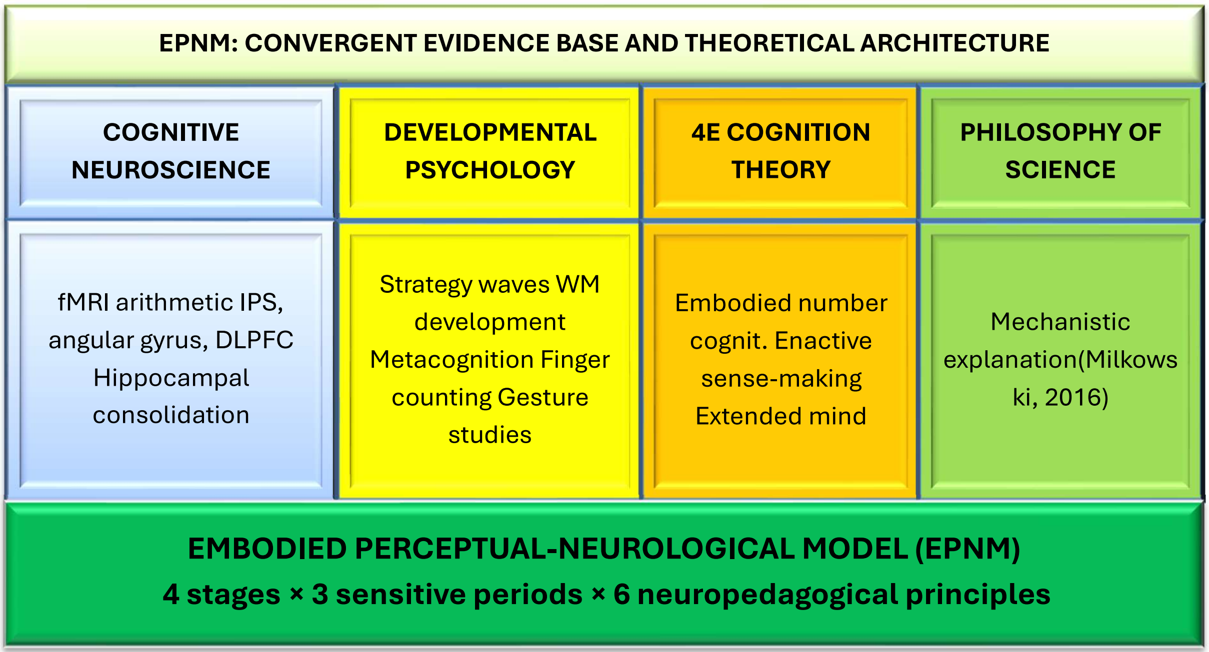

Figure 6. Convergent evidence base and theoretical architecture of the EPNM (IPS = intraparietal sulcus; DLPFC = dorsolateral prefrontal cortex; WM = working memory)

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